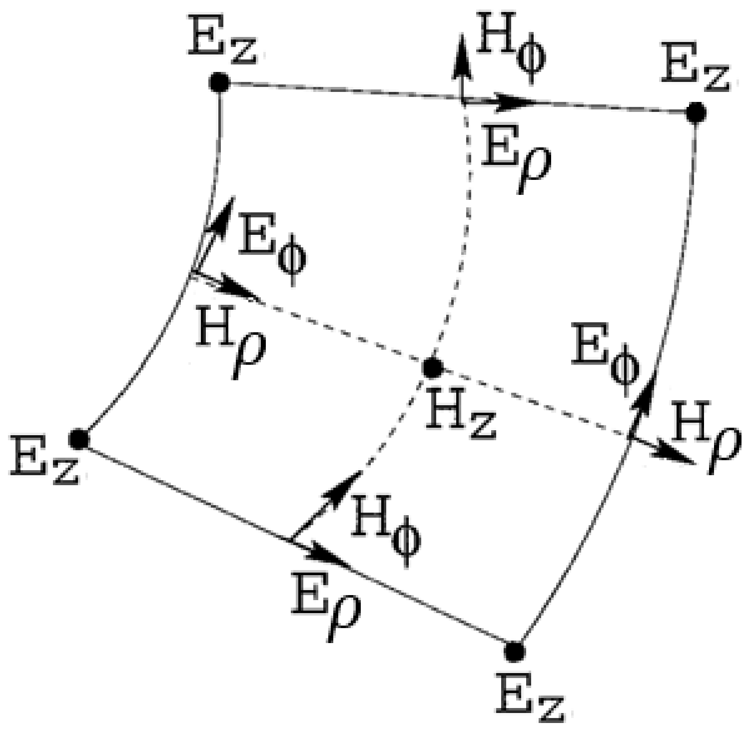
Passive and Active Circuit Elements in Cylindrical Finite-Difference Time-Domain Method



Abdullah M. Algarni, Atef Z. Elsherbeni, and Mohammed F. Hadi

ELECTRICAL ENGINEERING
COLORADO SCHOOL OF MINES

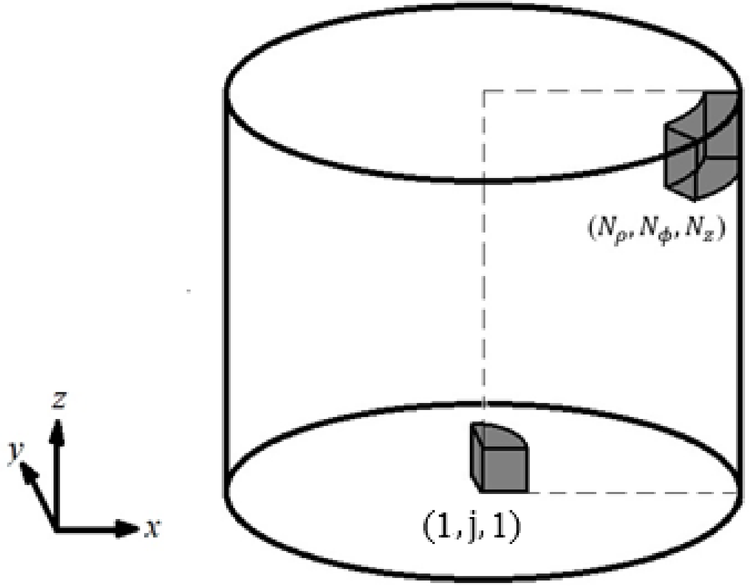
ARC
Antenna, RFID, and Computational EM Group

Introduction to Cylindrical FDTD

- Finite-difference time-domain (FDTD) is a technique which is used to solve Maxwell's equations in time domain EM by using numerical approximation of the derivatives, mainly in the cartesian coordinates system [1].

- For cylindrical structure problems such as well logging [3] in Petroleum Engineering, the finite difference time domain (FDTD) method in cylindrical coordinate is required

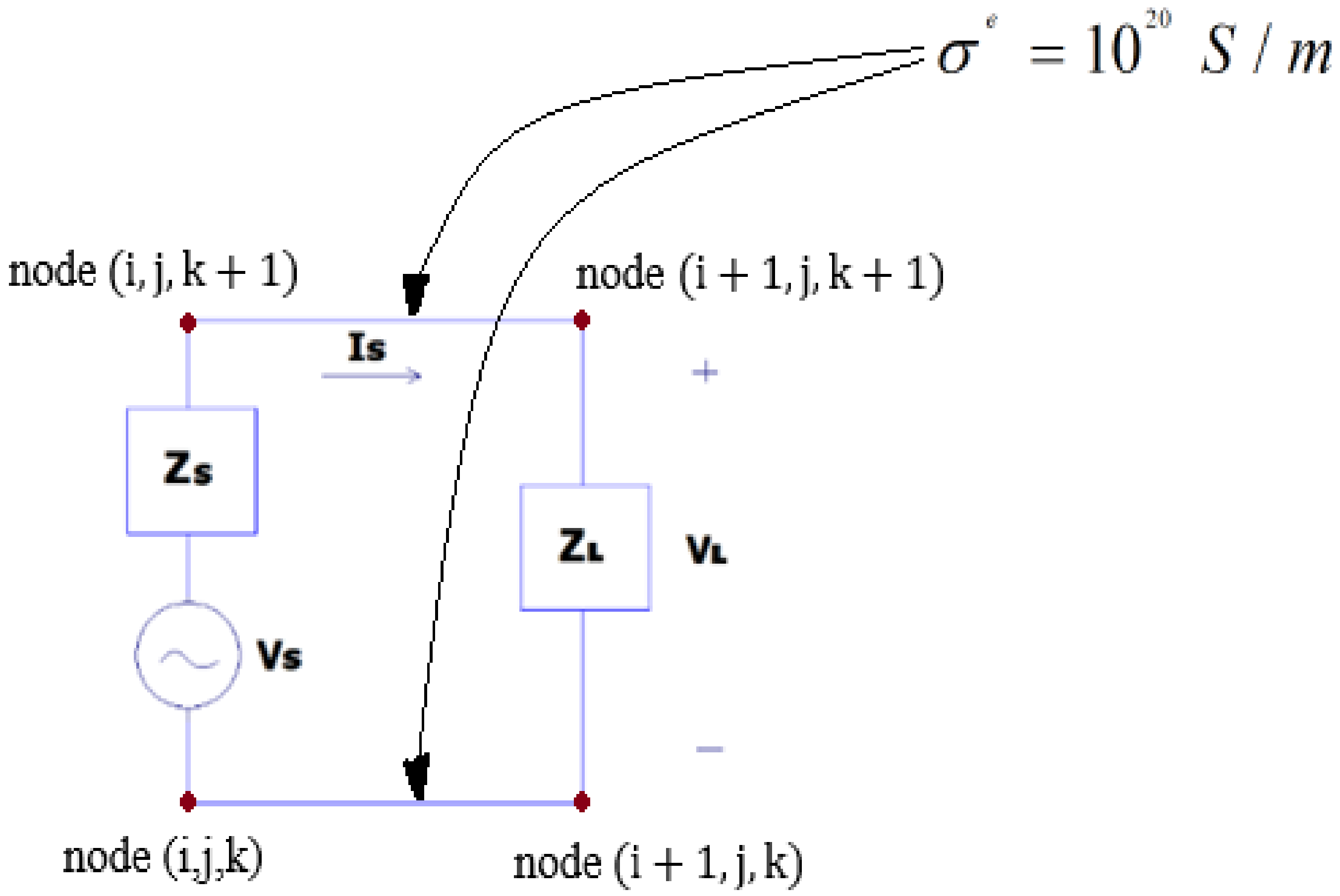
- The active and passive circuit elements can be used to design different type of sources such as dipole antenna. So, it is necessarily to derive updating equations for these elements in cylindrical coordinate.



Circuit Components

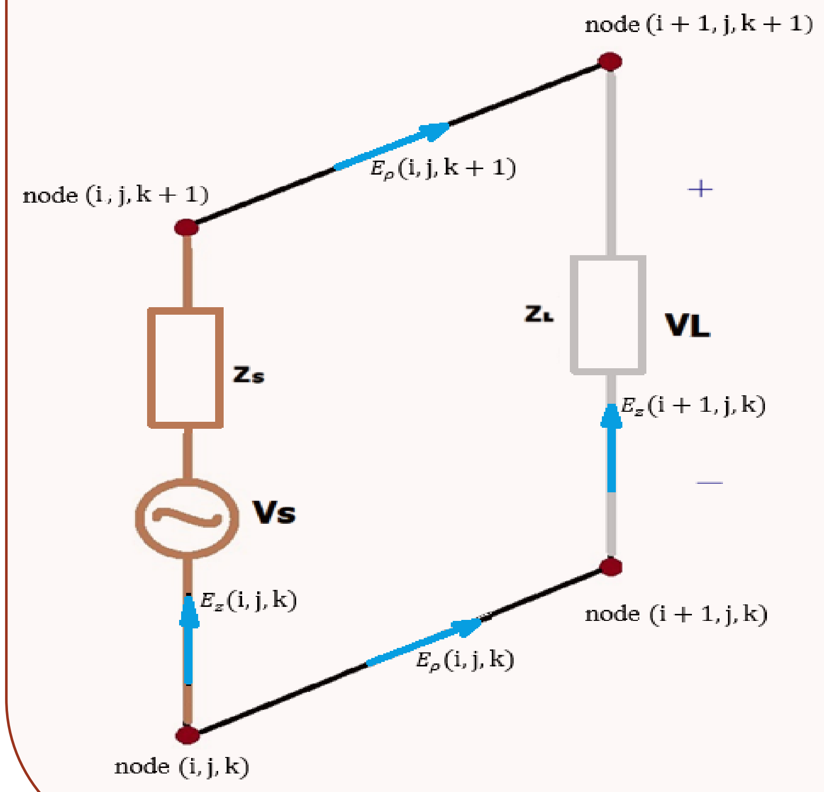
The simple circuit shown in the figure is implemented in the cylindrical FDTD (CFDTD) method.

- The Z_S and Z_L can be real or complex numbers.
- The waveform of the source V_S will be a Gaussian pulse to solve for a wide range of frequencies [1].
- The σ is the conductivity of the wire and it should be as high as possible to represent a perfect electric conductor.



Updating Equation in z-direction

The circuit elements can be placed between any two nodes in the CFDTD.



- For this case, the following electric field updating equation in the z-direction should be modified as follows:

$$
\begin{aligned}
E_z^{n+1}(i,j,k) = C_{eze}(i,j,k) \times E_z^{n}(i,j,k) \\
+ (C_{ezh\phi 1}(i,j,k) \times H_\phi^{n+\frac{1}{2}}(i,j,k) \\
- C_{ezh\phi 2}(i,j,k) \times H_\phi^{n-\frac{1}{2}}(i-1,j,k)) \\
+ C_{ezh\rho}(i,j,k) \times (H_\rho^{n+\frac{1}{2}}(i,j,k) - H_\rho^{n+\frac{1}{2}}(i,j-1,k)) \\
+ C_{ezj}(i,j,k) \times J_{iz}^{n+\frac{1}{2}}(i,j,k),
\end{aligned}
$$

where, $C_{ezj}(i,j,k) = -\dfrac{2\Delta t}{2\varepsilon_z(i,j,k) + \Delta t \sigma_z^e(i,j,k)}$,

- The other coefficients can be expressed from (3) in [2].

- The E_z is then modified with the new definition of $J_{iz}^{n+\frac{1}{2}}(i,j,k)$

ELECTRICAL ENGINEERING
COLORADO SCHOOL OF MINES

ARC

Updating Equations for Passive and Active Elements

- In this work, the circuit elements at the source side is one of three combinations. The first combination is a source with a resistor only, the second combination is a source with a resistor and a capacitor in series, and the third combination is a source with a resistor and an inductor in series.
- The load side is a single element either a resistor, a capacitor, or an inductor [1].
- For the three cases in the source side, the current density in the updating equation can be represented as following:

For case 1: $Z_s = R_s$, $$J_{iz}^{n+\frac{1}{2}}(i,j,k) = \frac{\Delta z}{2A_i R_S}\left(E_z^{n+1}(i,j,k) + E_z^{n}(i,j,k)\right) + \frac{V_S}{A_i R_S},$$

For case 2: $Z_s = R_s + X_L$, $$J_{iz}^{n+\frac{1}{2}}(i,j,k) = \frac{2L - R_S\Delta t}{2L + R_S\Delta t} J_{iz}^{n-\frac{1}{2}}(i,j,k) + \frac{2\Delta z\Delta t}{A_i(2L + R_S\Delta t)} E_z^{n}(i,j,k) + \frac{2\Delta z\Delta t}{A_i(2L + R_S\Delta t)} V_S,$$

and, for case 3: $Z_s = R_s + X_C$, we used the recursive convolution method

$$Q_{iz}^{n+\frac{1}{2}}(i,j,k) = Q_{iz}^{n-\frac{1}{2}}(i,j,k) \times e^{-\frac{\Delta t}{CR_S}} + \left(\Delta z E_z^{n}(i,j,k) + V_S\right),$$

$$J_{iz}^{n+\frac{1}{2}}(i,j,k) = \frac{1}{A_i R_S}\left(\Delta z E_z^{n}(i,j,k) + V_S\right) + \left(e^{-\frac{\Delta t}{CR_S}} - 1\right) \times Q_{iz}^{n+\frac{1}{2}}(i,j,k),$$

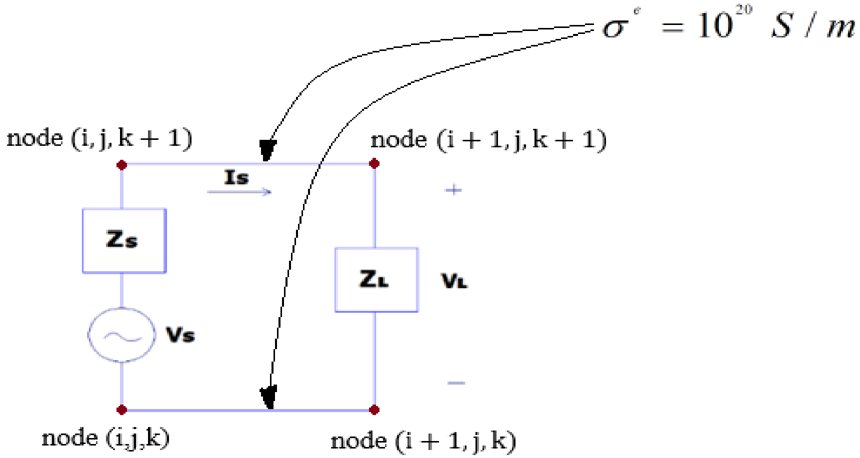
The Simulation Parameters

- In this work, the following configuration will be implemented with

$$Z_s = R_s + X_L$$
$$Z_L = R_L$$

- where

$$R_s = R_L = 50\Omega$$
$$L = 50 \text{ pH}$$



- The following parameters are used in the simulation:

Domain size:

$$N_\rho = 11,\ N_\phi = 9, \text{ and } N_z = 10,$$

The step size:

$$\Delta\rho = \Delta z = 0.1873 \text{ mm}, \text{ and } \Delta\phi = \frac{2\pi}{9} \text{ rad},$$

The time step size and the number of time step size:

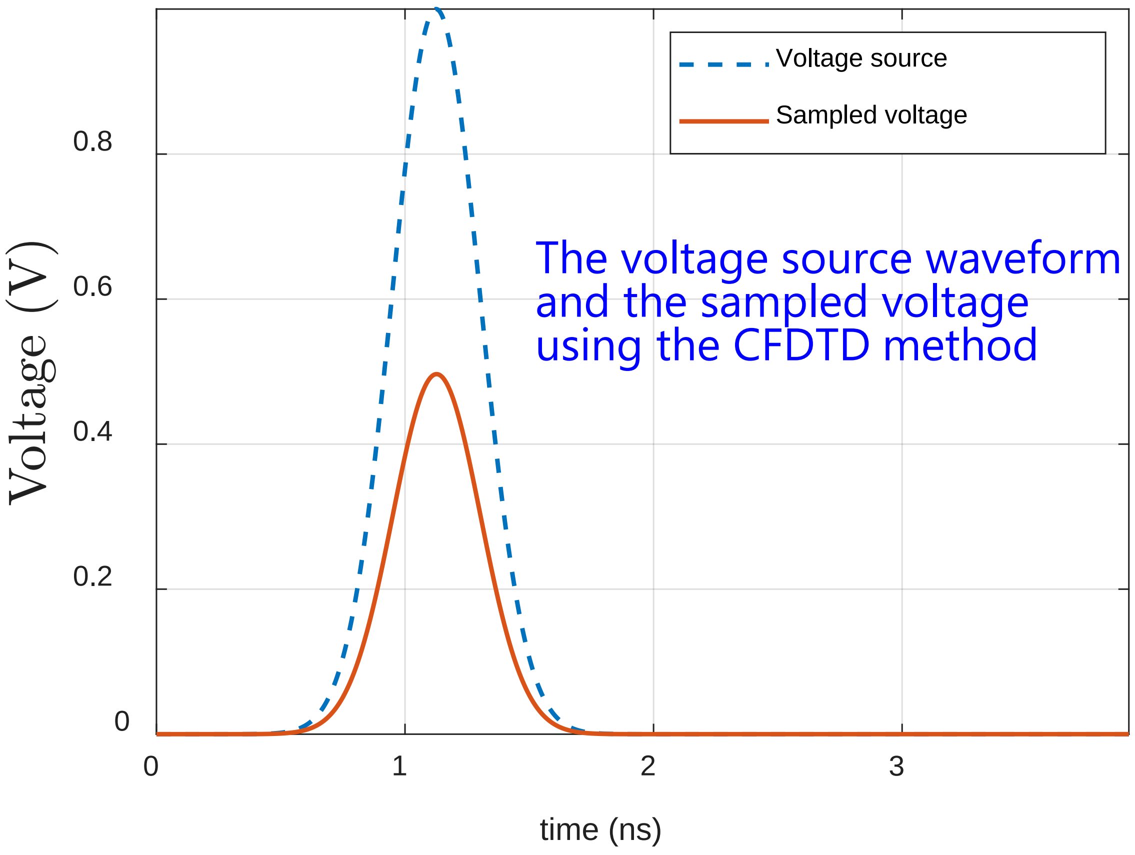
$$\Delta t = 1.9563 \times 10^{-13} \text{ second}, \text{ and } N = 20000.$$

The Voltage Source and Sampled Voltage at the Load

- To compare the result from the simulation, the following theoretical equation is used:

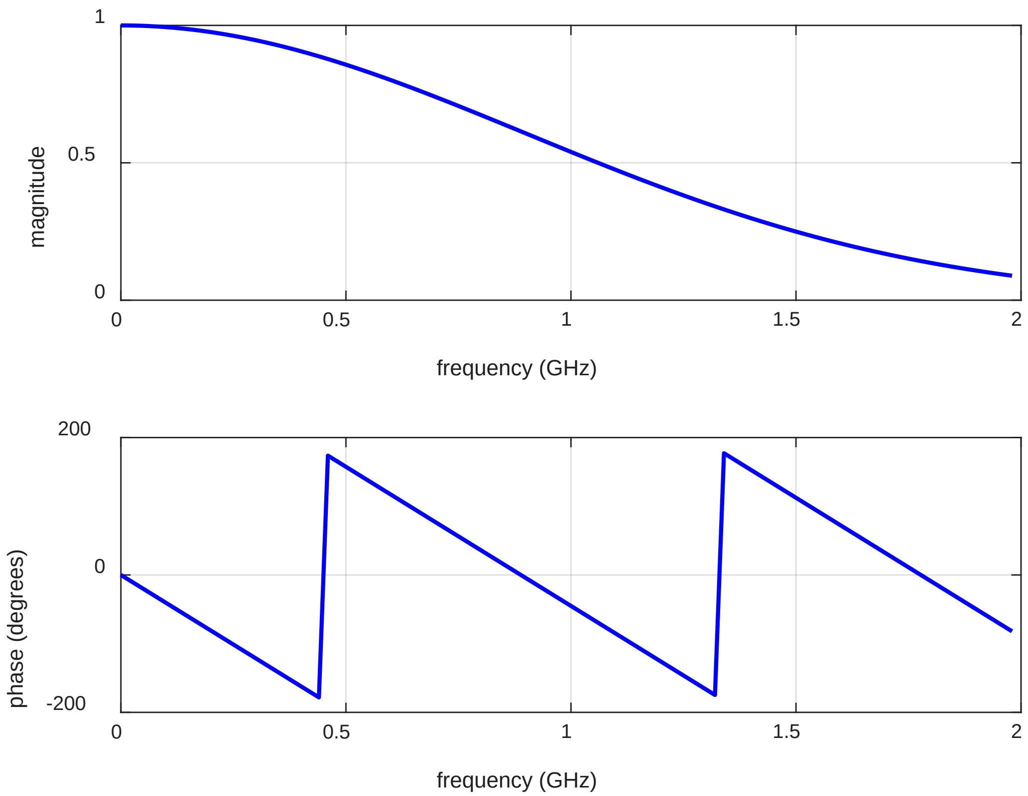
$$V_L = \left(\frac{R_L}{R_L + R_s + j\omega L} \right) V_s$$

- The transforming to frequency domain for the voltage source and sampled voltage must be done first to compare the results with the theoretical equation .

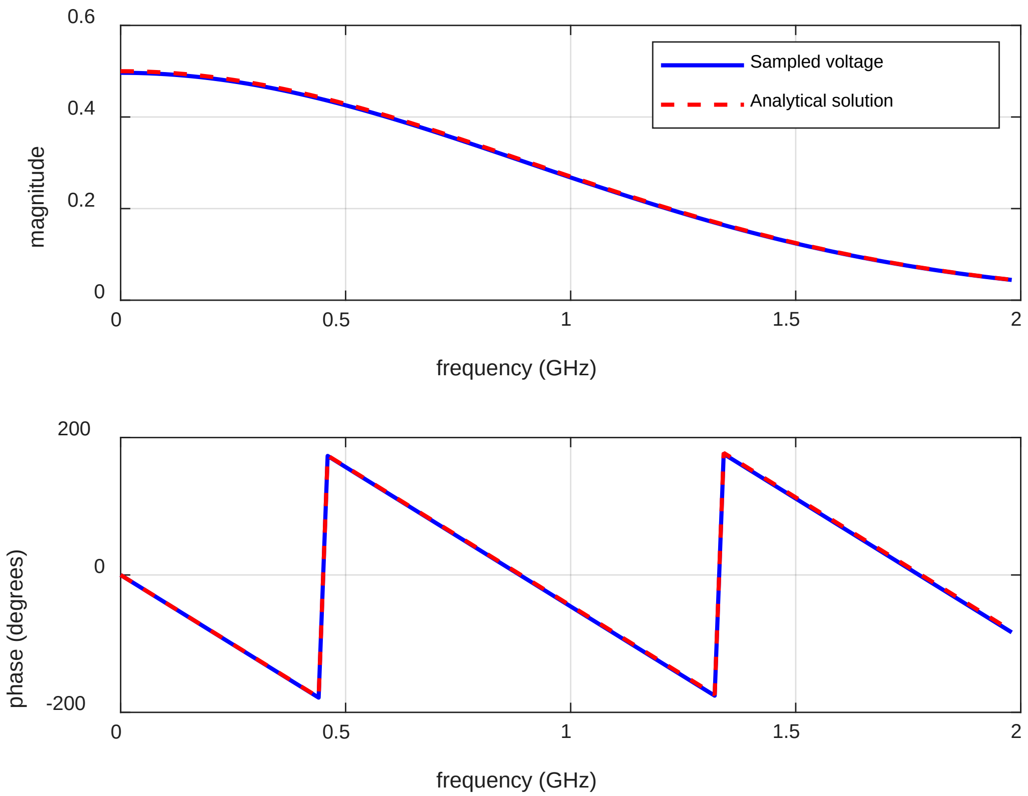


CFDTD Results

- The voltage source in the frequency domain:

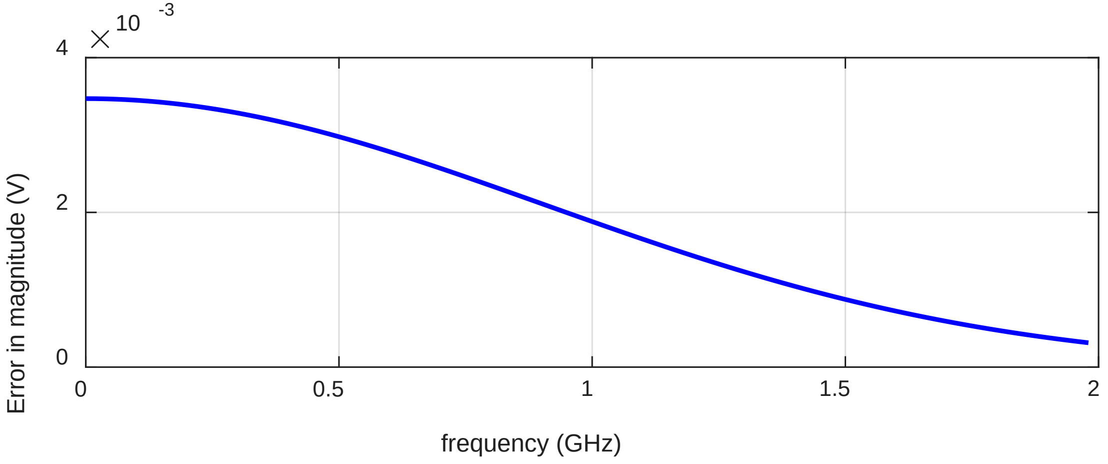


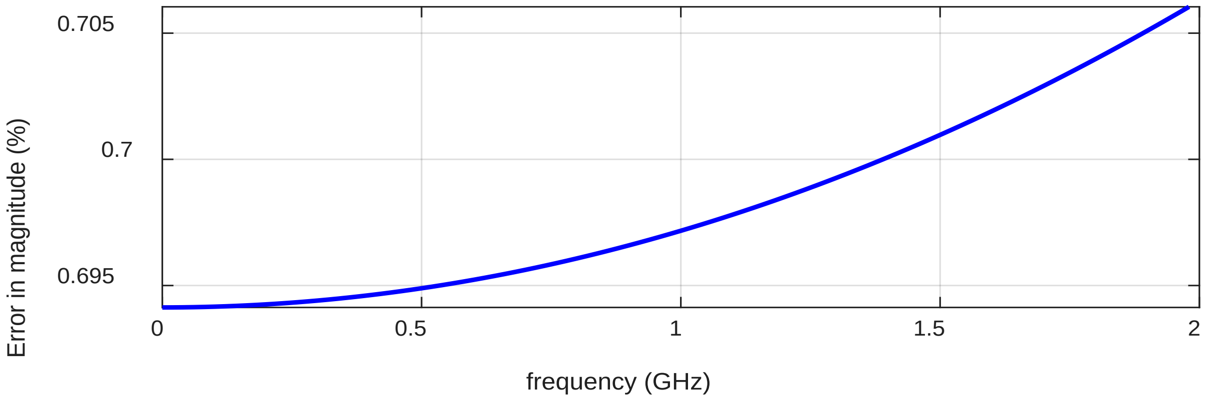
- The sampled voltage vs. the theoretical results in the frequency domain:

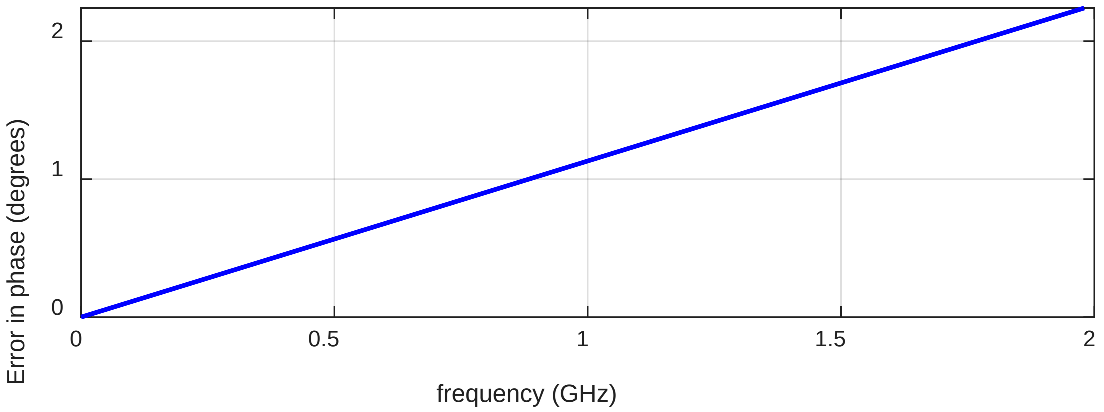


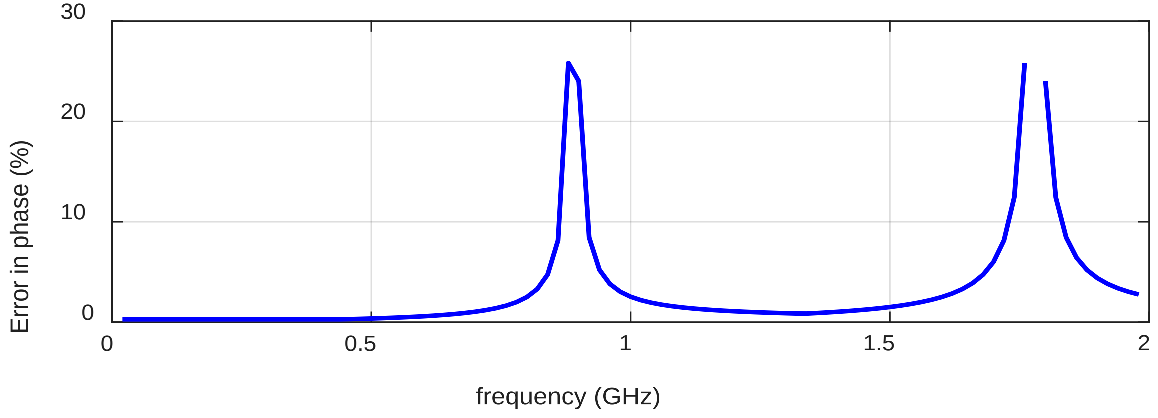
CFDTD Error Analysis

The absolute and percentage errors between sampled voltage and the theoretical results.









Conclusions and References

- The validation of integrating passive and active circuit elements into the CFDTD method and simulations is achieved.
- Work is in progress to include nonlinear elements in this cylindrical formulation to address a larger array of electromagnetic problems.

[1] A. Elsherbeni and V. Demir, The Finite Difference Time Domain Method For Electromagnetics with MATLAB® Simulation, 2nd ed., SCITECH Publishing, 2015.

[2] M. F. Hadi and A. Z. Elsherbeni "Numerical Dispersion and Stability for Three- Dimensional Cylindrical FDTD near the Axis of Rotation," 11th European Conference on Antennas and Propagation (EUCAP), pp. 936-938, May 2017.

[3] D. V. Ellis and J. M. Singer, Well Logging for Earth Scientists, 2nd ed., Springer, pp. 3, 2007.