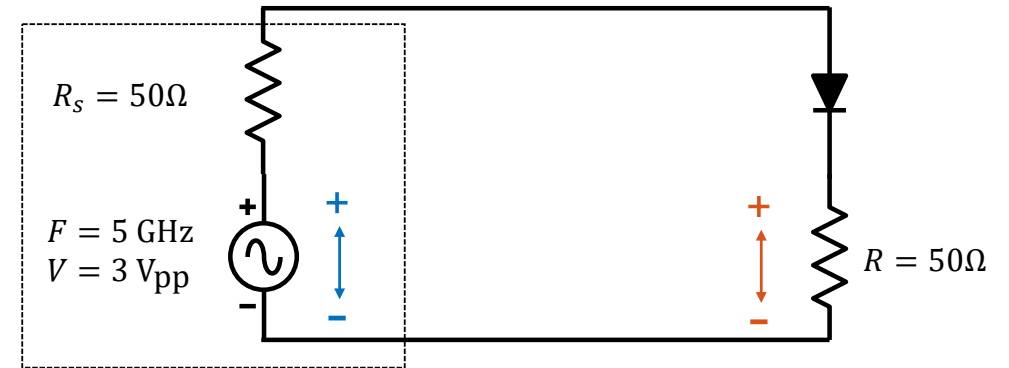


# Simulation of a Nonlinear Frequency Multiplier using the FDTD Technique

Joshua M. Kast and Atef Z. Elsherbeni

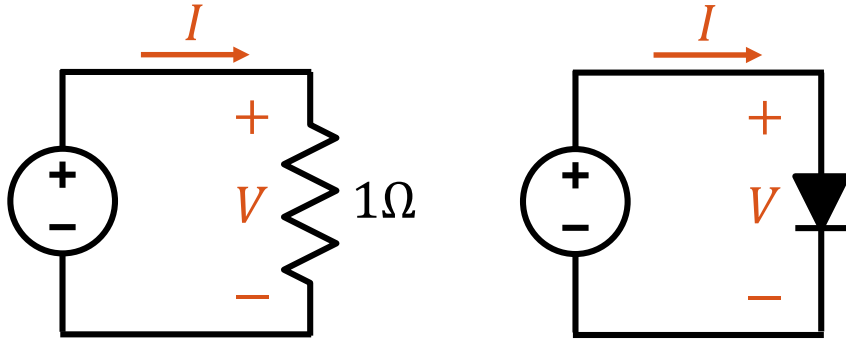
[jkast@mines.edu](mailto:jkast@mines.edu) [aelsheerb@mines.edu](mailto:aelsheerb@mines.edu)



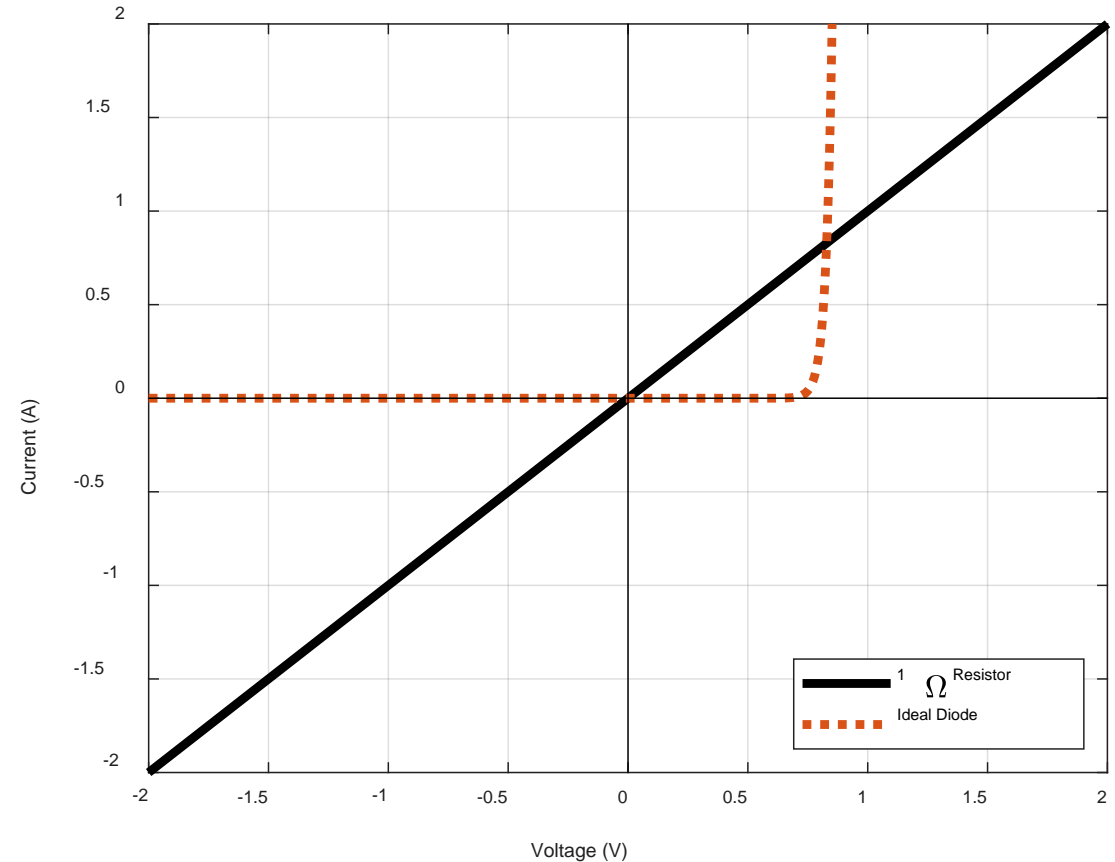
# Introduction

- Common RF devices, such as mixers and detectors, employ nonlinear components to function.
- Increasingly, nonlinear effects are employed to improve energy efficiency and thermal properties of modern amplifiers.
- Simulation is challenging with nonlinear devices – frequency-domain approaches break down.
- Goal: use nonlinear lumped-element devices integrated in FDTD grid to demonstrate nonlinear effects relevant to RF communications.

# Introduction - Nonlinearity



- Linear devices include resistors, inductors, capacitors.
- Nonlinear behavior occurs in diodes, transistors, and ferrites.

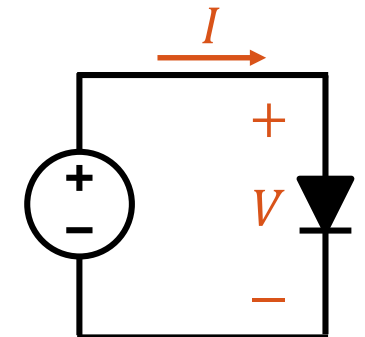
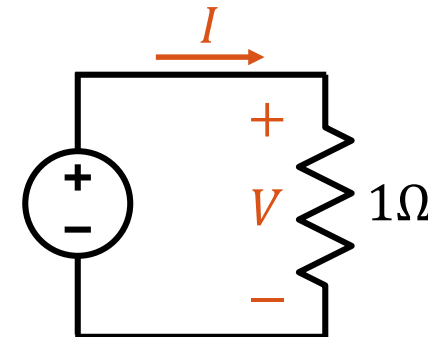
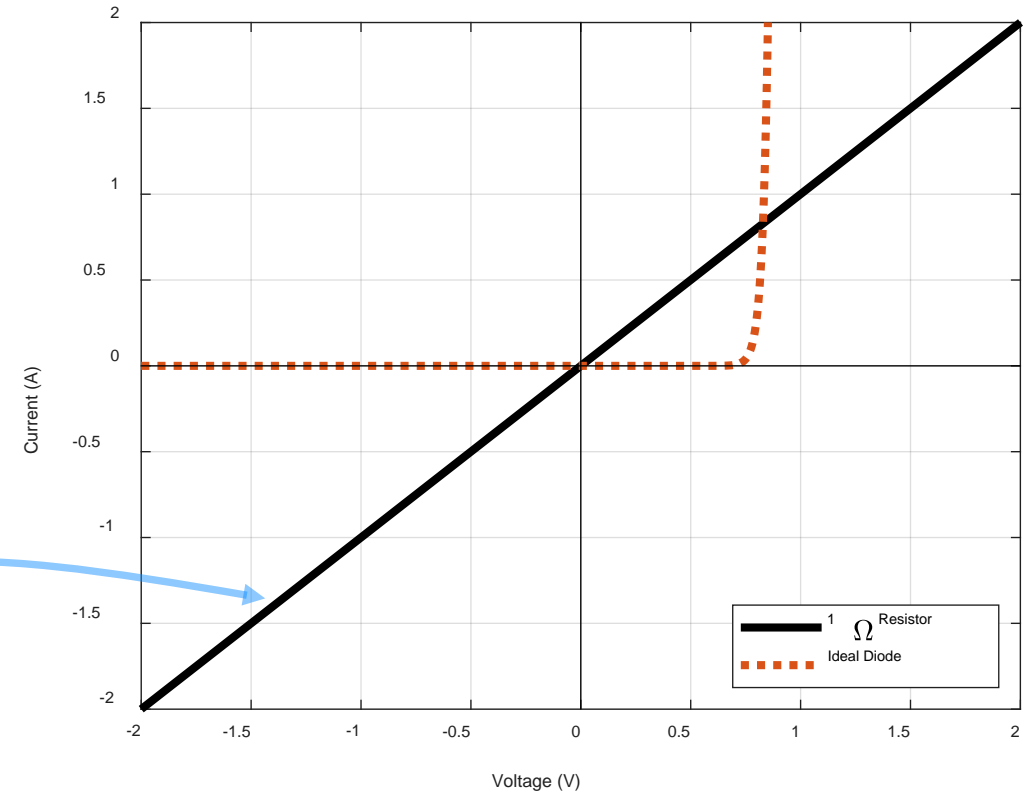


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- Example: resistor and diode
  - I/V characteristic for resistor is defined by Ohm's law:

$$I = \frac{V}{R}$$

Linear Equation



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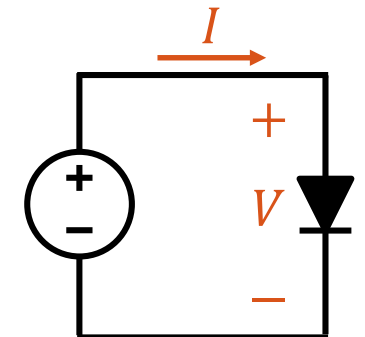
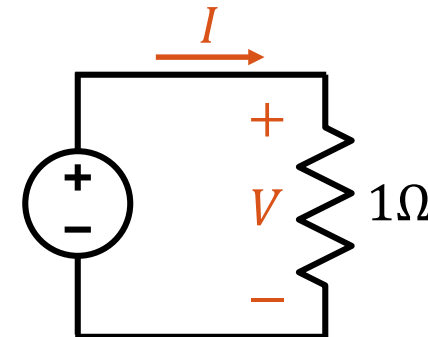
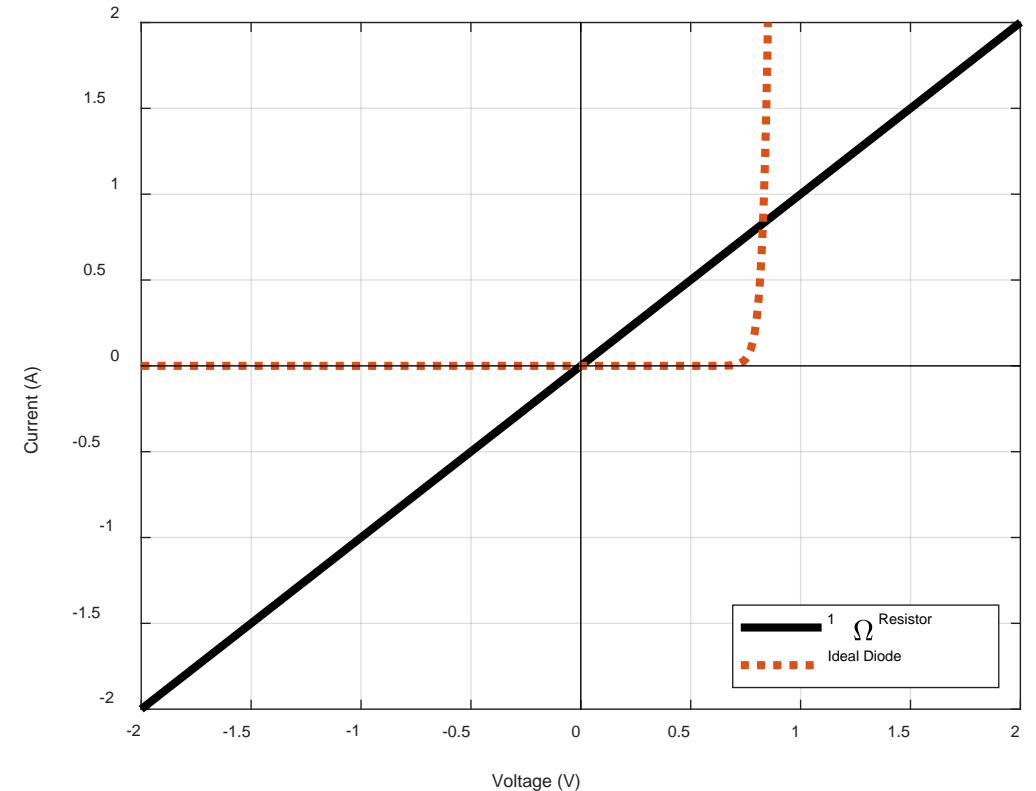
- I/V characteristic for resistor is defined by Ohm's law:

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- I/V characteristic for diode is defined by an exponential function:

$$I_D = I_S \left[ e^{\frac{V_D}{\eta V_T}} - 1 \right]$$

Exponential Equation



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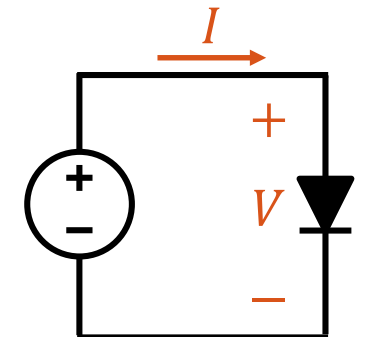
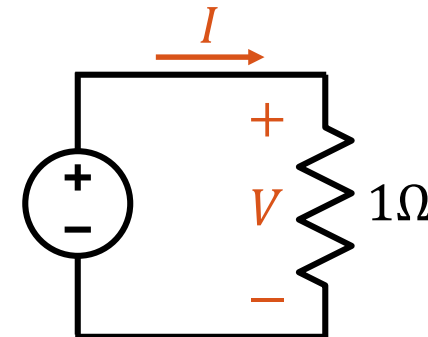
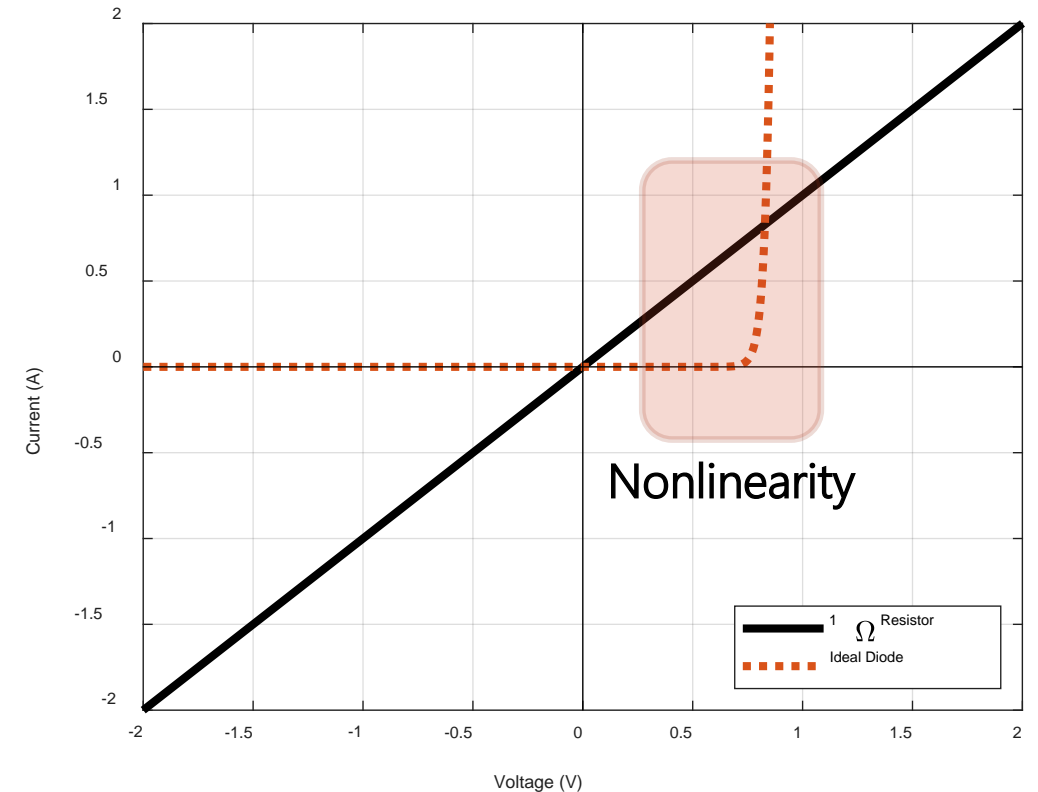
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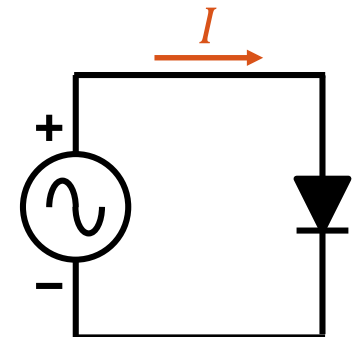
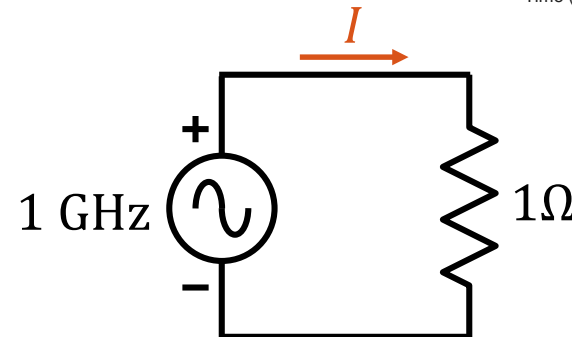
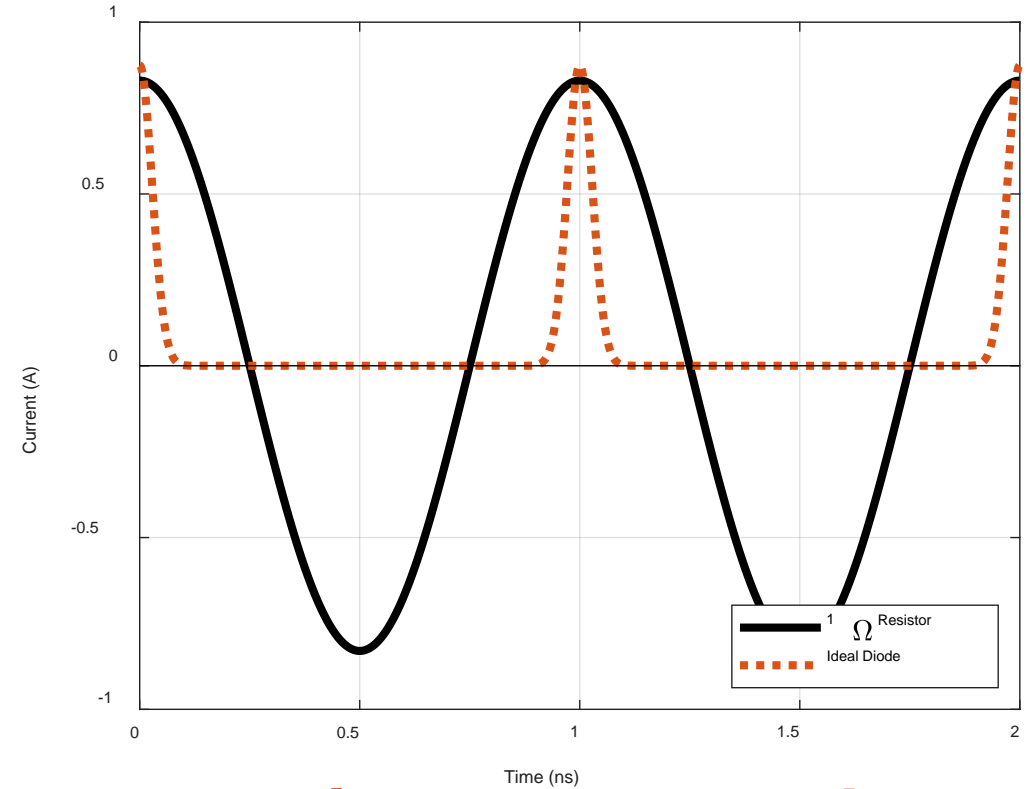


# Nonlinearity in The Frequency Domain

- Example: Excite a diode with a sinusoidal voltage source, measure current.

$$V_D(t) = a \cos(\omega t) \quad \text{Substitute} \rightarrow \quad I_D = I_S \left[ e^{\frac{V_D}{\eta V_T}} - 1 \right]$$

$$I_D(t) = I_S \left[ e^{\frac{a \cos(\omega t)}{\eta V_T}} - 1 \right]$$



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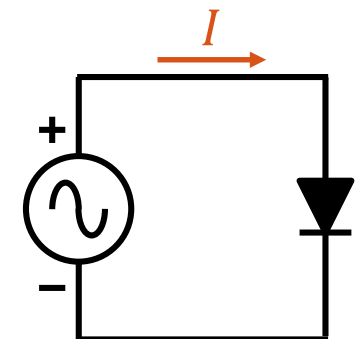
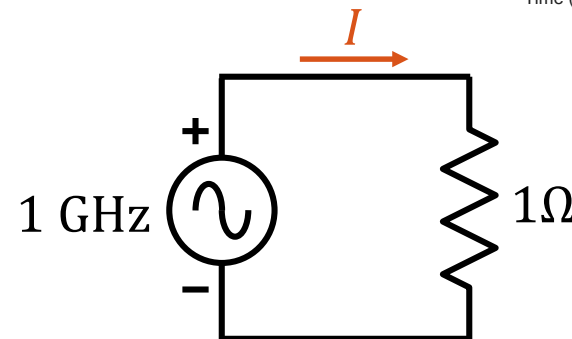
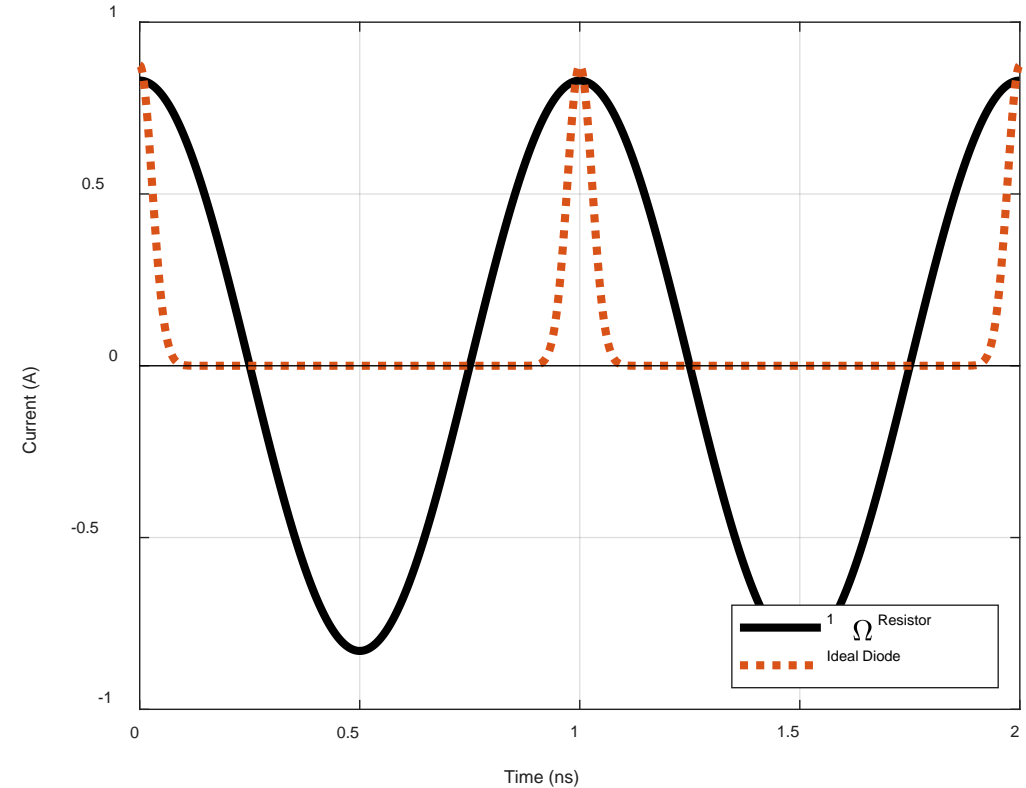
$$I_D(t) = I_S \left[ e^{\frac{a \cos(\omega t)}{\eta V_T}} - 1 \right]$$

Replace exponential by  
Maclaurin series:

$$I_D(t) = I_S \sum_{n=1}^{\infty} \frac{\left( \frac{a \cos(\omega t)}{\eta V_T} \right)^n}{n!}$$

Take first three terms:

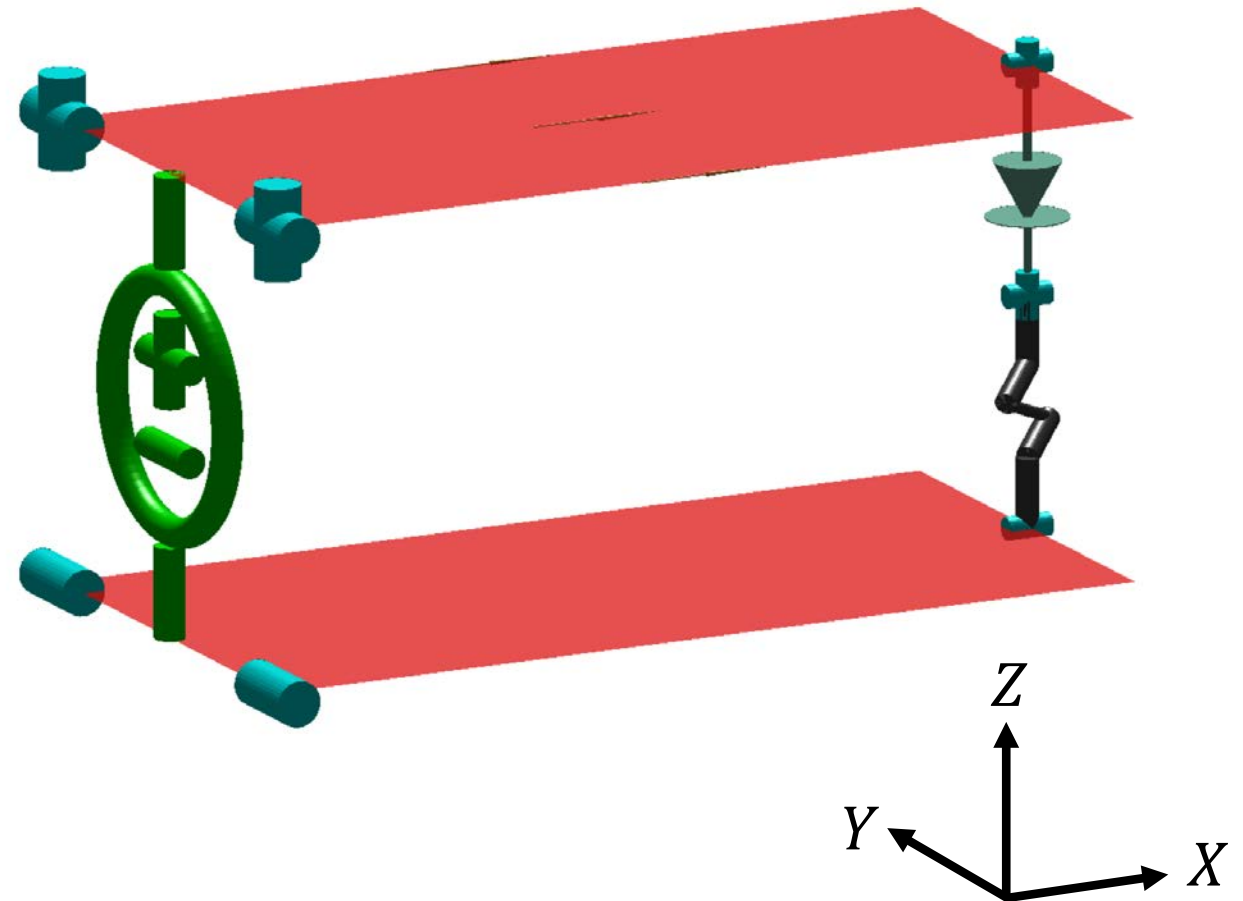
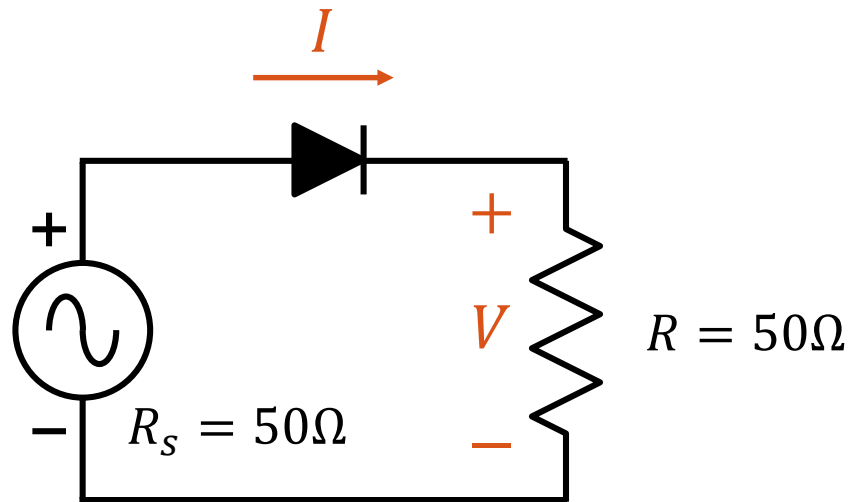
$$I_D(t) \approx I_S \left( \frac{a \cos(\omega t)}{\eta V_T} + \frac{a^2 (\cos(2\omega t) + 1)}{4\eta^2 V_T^2} + \frac{a^3 (\cos(3\omega t) - 3 \cos(\omega t))}{24\eta^3 V_T^3} \right)$$



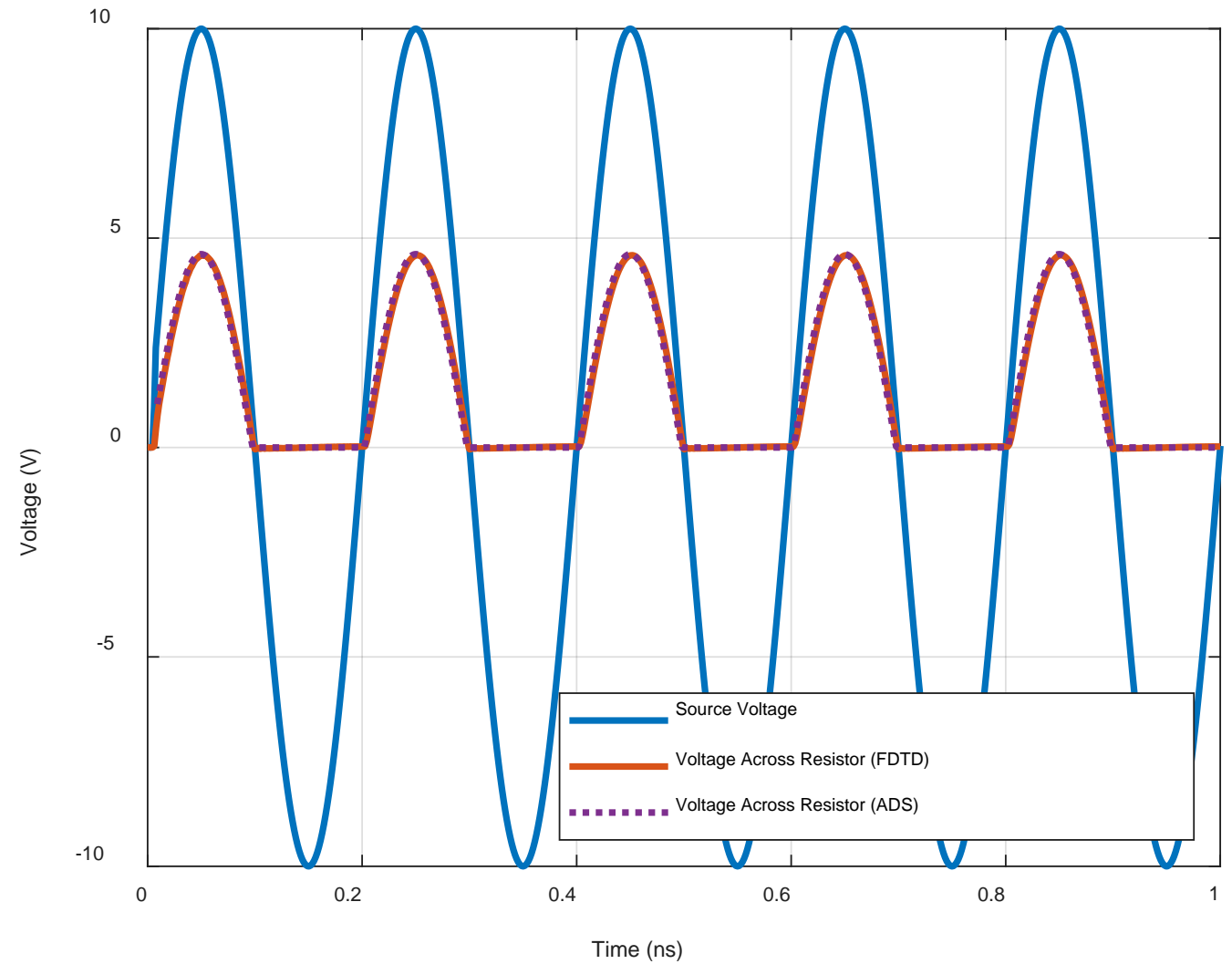
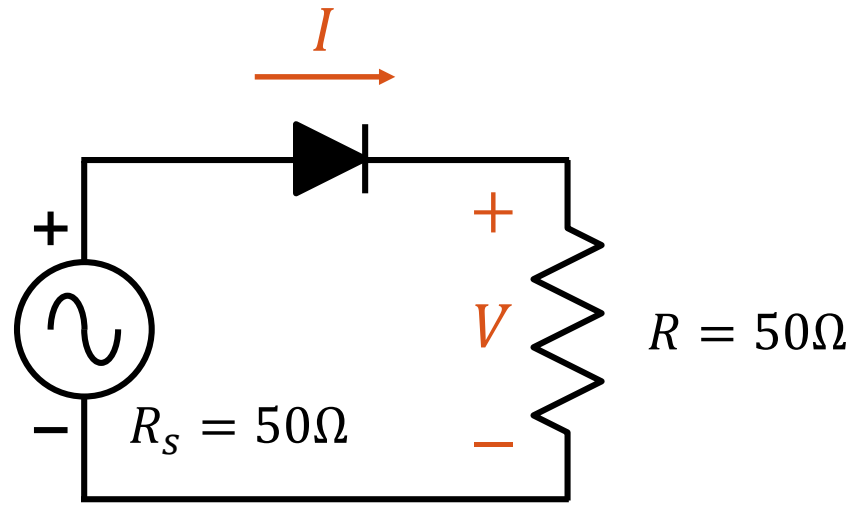
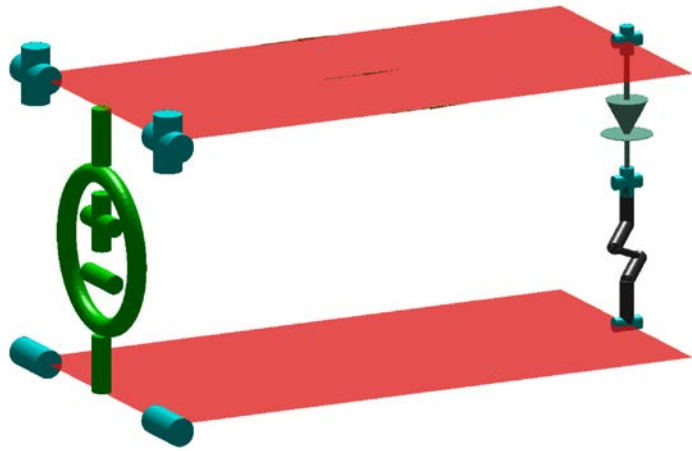


# Diode With Series Resistor

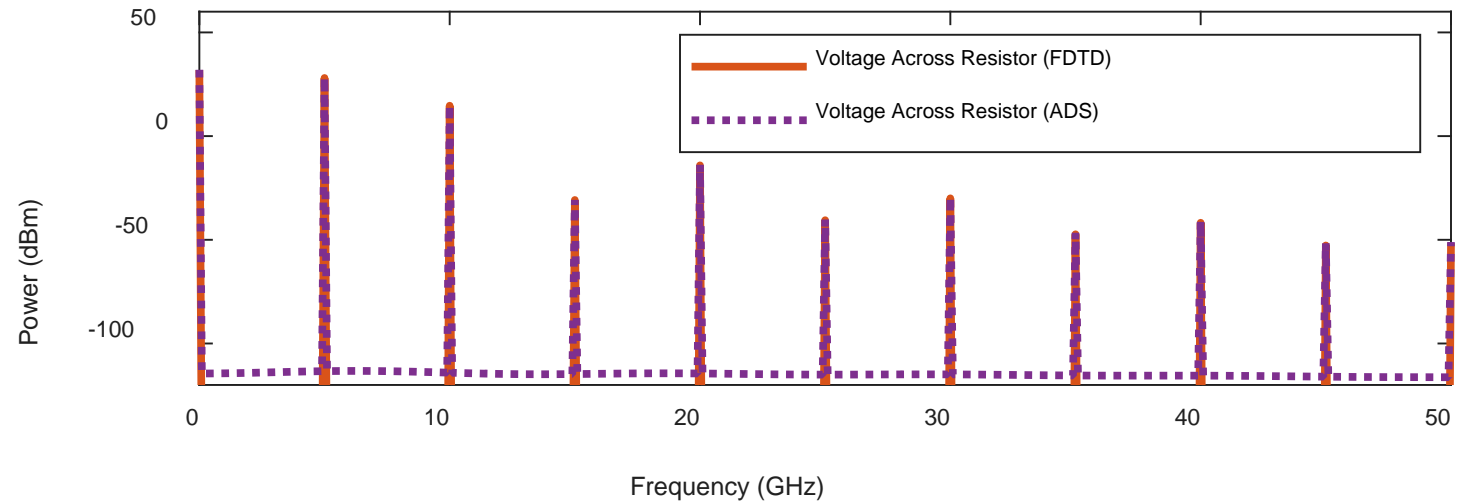
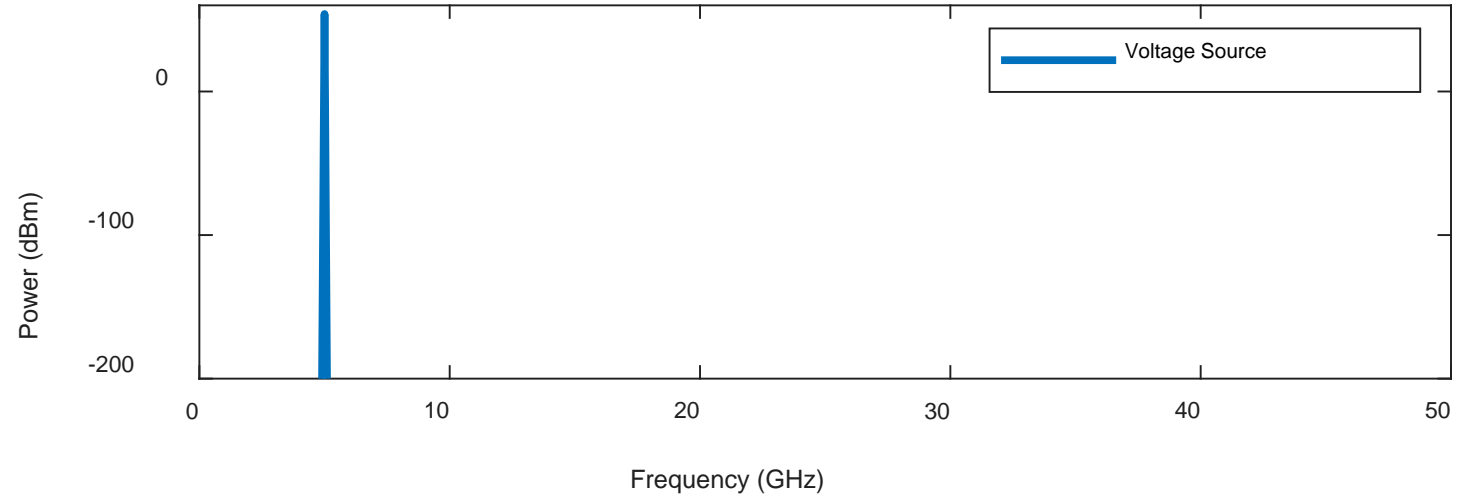
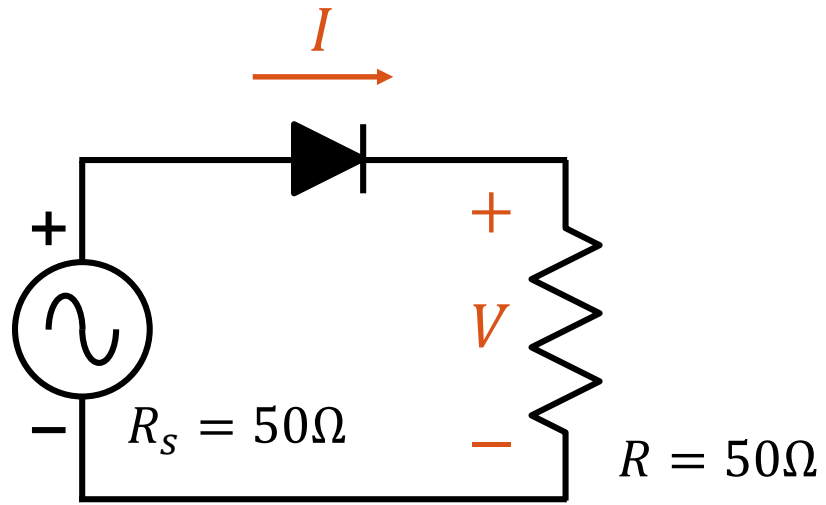
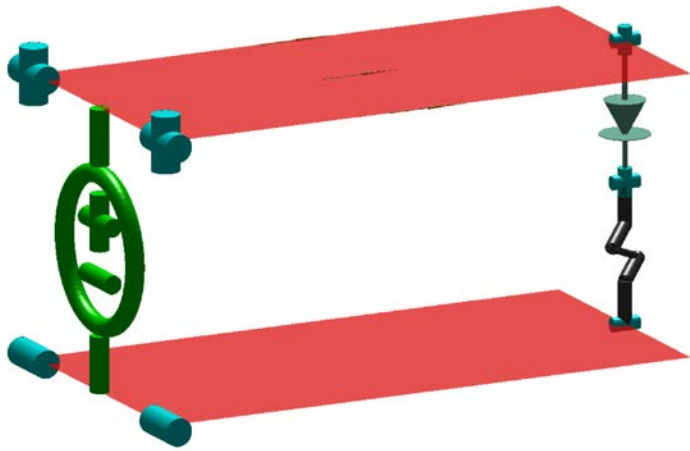
- FDTD Domain: 40 x 38 x 43 (65360) cells
  - CPML Boundaries – 10 cells with 8 cell air-buffer
  - Cell size -  $dx = dy = dz = 0.05$  mm
- Excitation:  $10 V_{pp}$ , 5 GHz sinusoidal



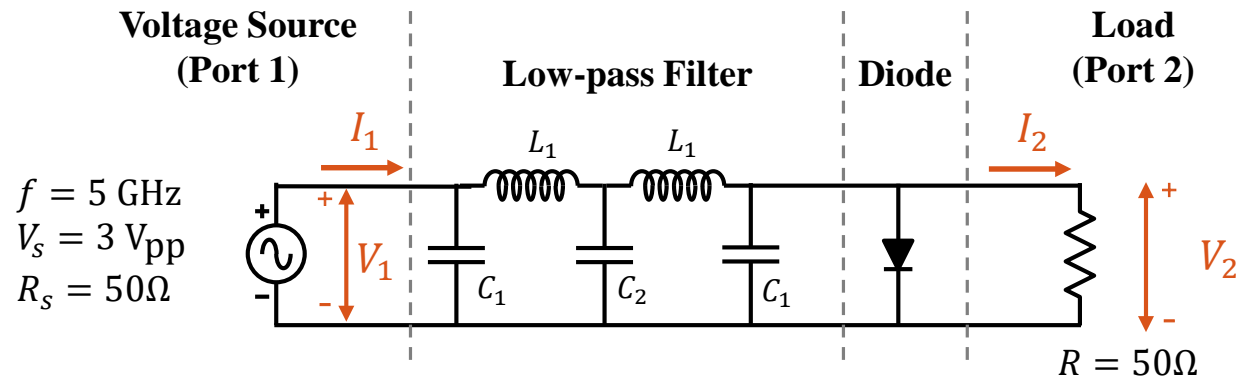
# Diode With Series Resistor: Time-Domain Results



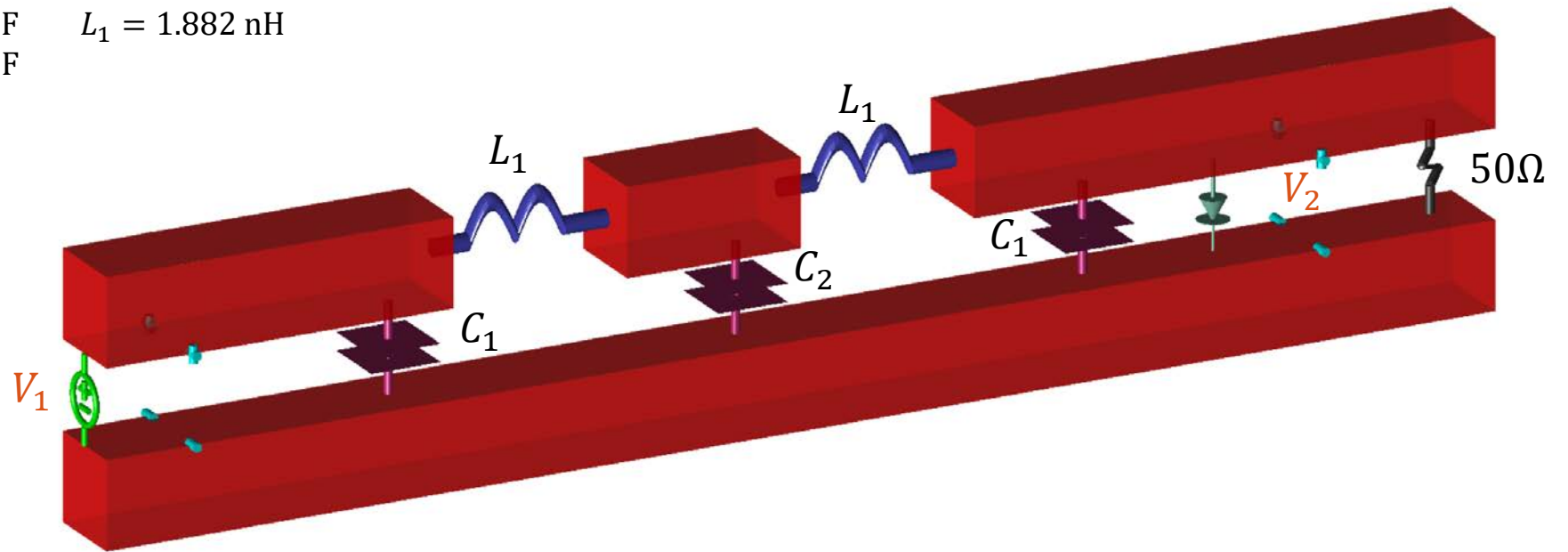
# Diode With Series Resistor: Frequency Domain Results



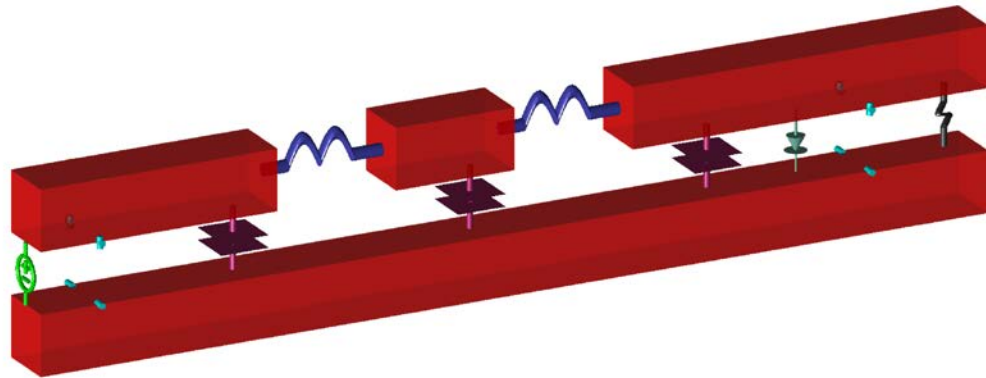
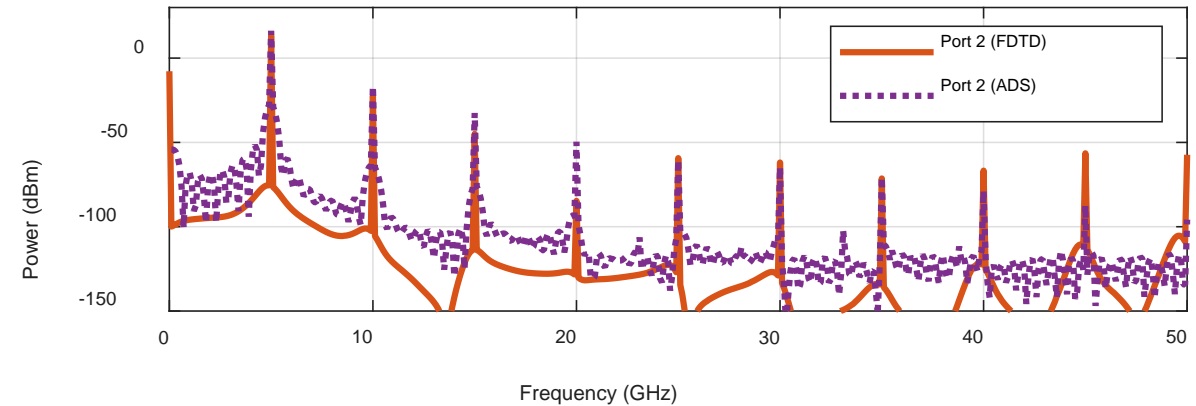
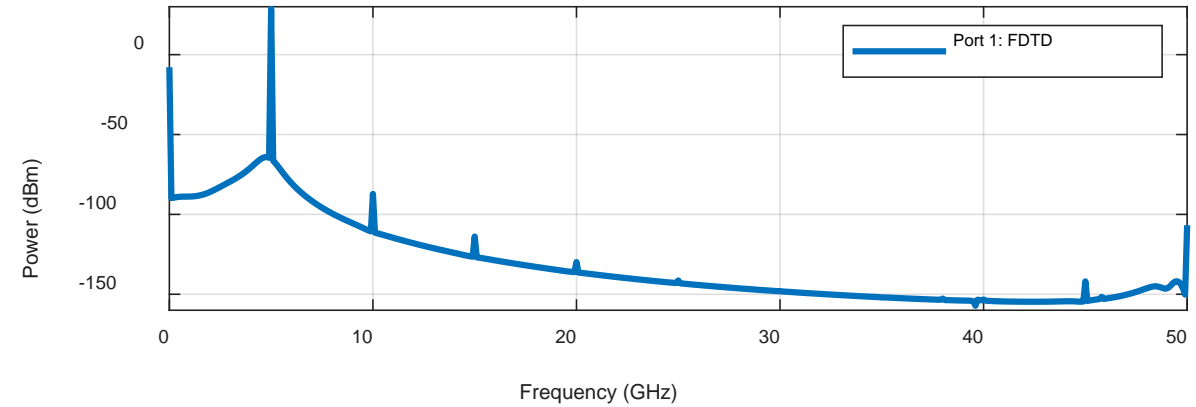
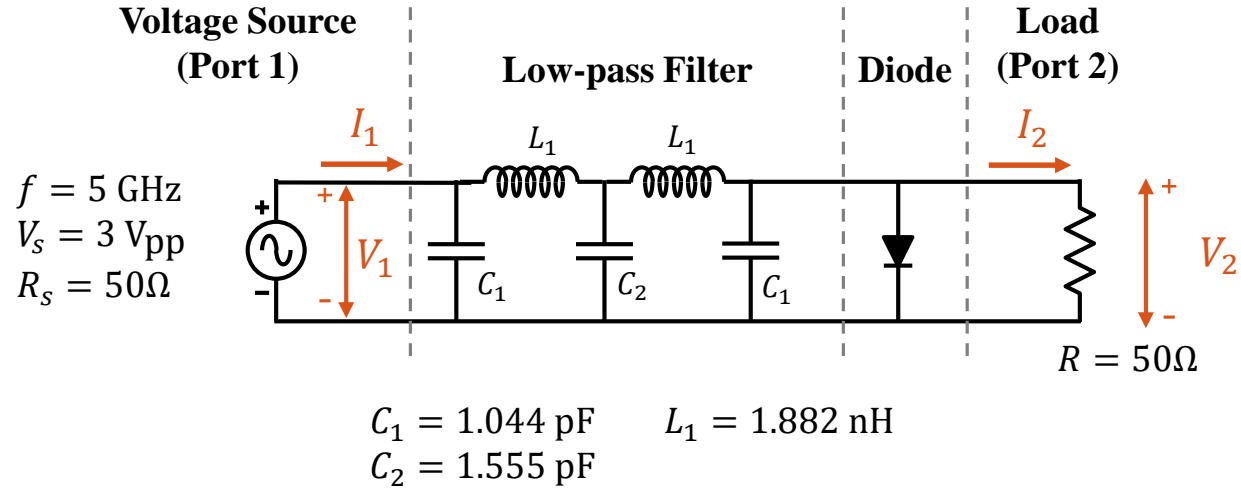
# Diode with Low Pass Filter



$C_1 = 1.044 \text{ pF}$      $L_1 = 1.882 \text{ nH}$   
 $C_2 = 1.555 \text{ pF}$

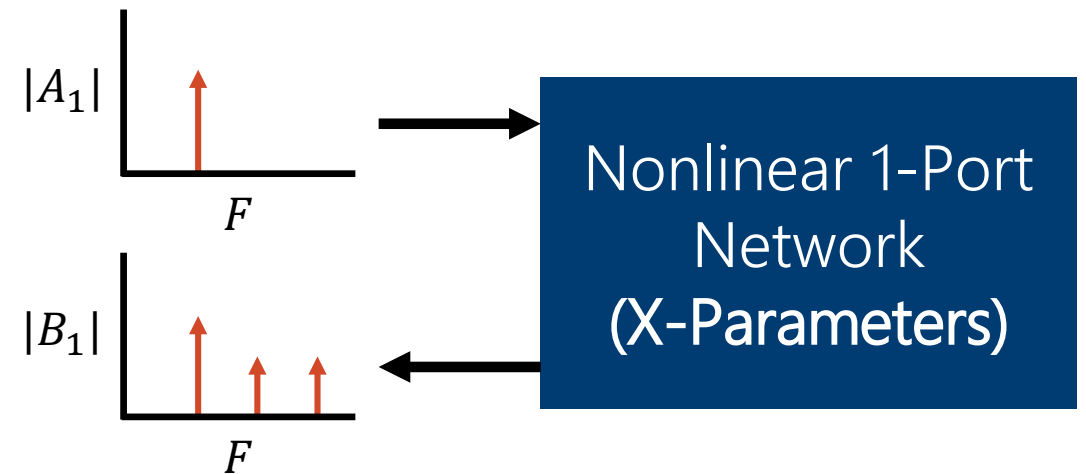


# Diode with Low Pass Filter



# Conclusions and Future Work

- FDTD provides useful simulations of nonlinear components integrated in microwave circuits.
- Future work: analyze the results using the nonlinear X-parameters



# References

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- [4] Keysight, Santa Rosa, CA, USA. 2019. Advanced Design System 2019 [Online]. Available: <http://www.keysight.com/en/pc-1297113/advanced-design-system-ads?&cc=US&lc=eng>
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- [6] K. ElMahgoub and A. Z. Elsherbeni, "FDTD Implementations of Integrated Dependent Sources in Full- Wave Electromagnetic Simulations," vol. 29, no. 12, p. 10, 2014