Sub-Gridding Errors in Standard and Hybrid Higher Order FDTD Simulations

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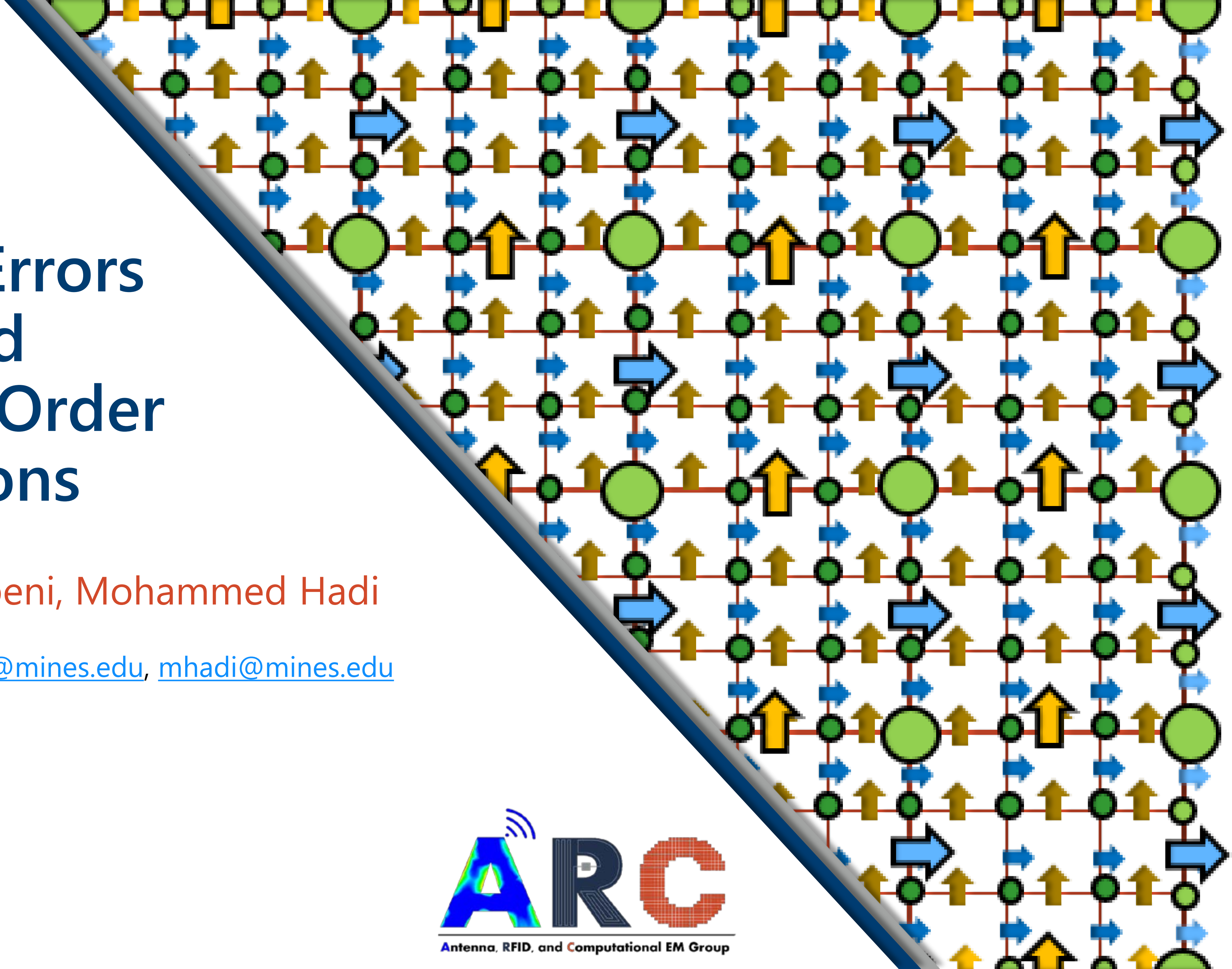

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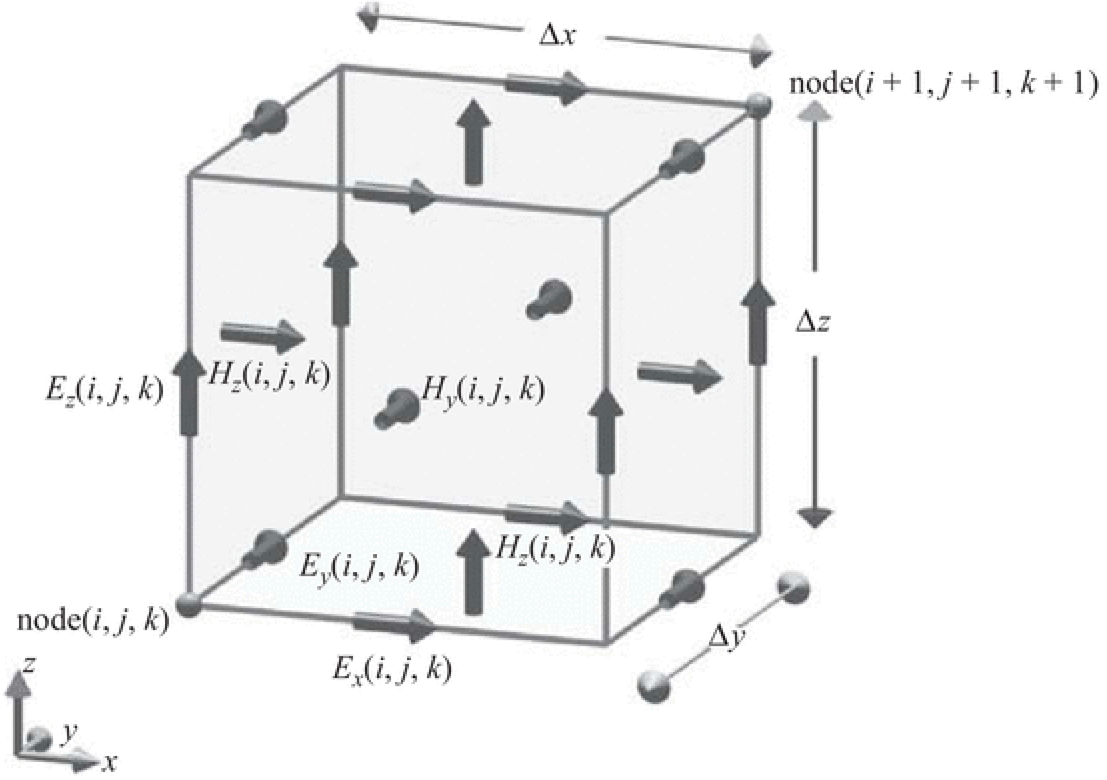
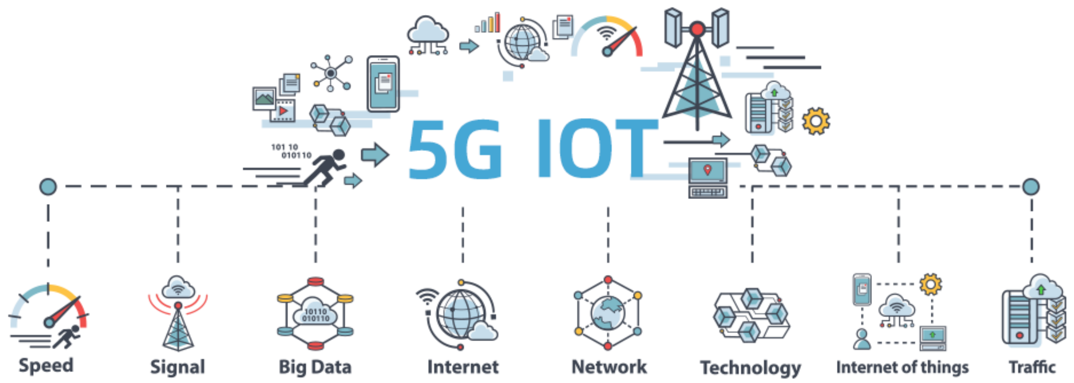
Introduction

- What is the **Finite-Difference Time-Domain Method**(FDTD)?
- What is it used for?
- Why do we need Subgridding?



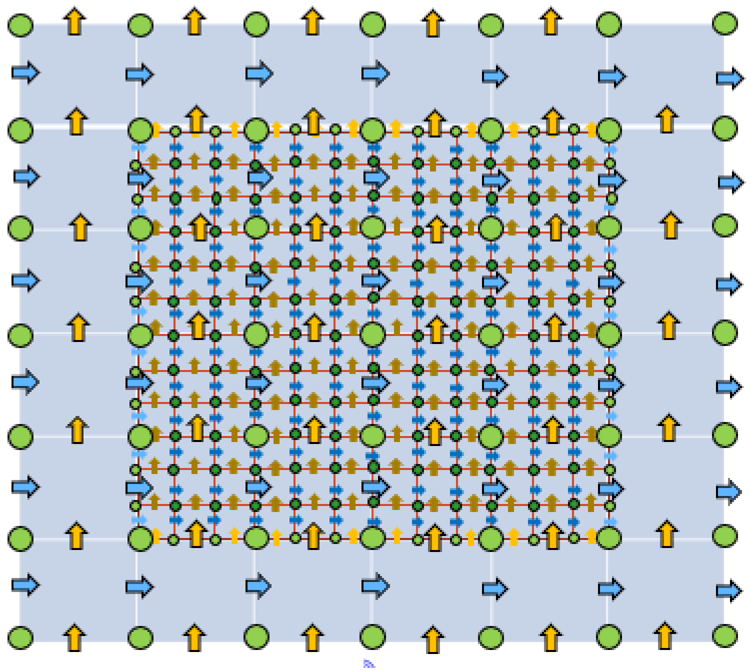
Research Focus

- **Problem:** Difficulties with **5G and IoT** Device Design



https://www.ursalink.com/en/blog/5g-iot

- **Proposed Solution:** Subgridding can be used to save memory and CPU time while maintaining an accurate solution.



Main Research Objectives

- The repercussions of subgridding in a FDTD calculation can lead to **dispersion** and **stability errors** [1-3].
- A larger subgrid enhances the maximum area an object of interest can be meshed to receive a more accurate analysis in a local grid. The deleterious effects of larger subgridding ratios have been discussed in the literature [4].
- A topic that has not yet been investigated, is the relative error that arises with increased electrical sizes of subgridded regions, independent of the contrast ratio.
- This research will focus on the effect the size of a subgridded region has on the resulting errors with **1:3, 1:9, 1:15, and 1:27 contrast ratios** within 1D and 2D FDTD simulations.

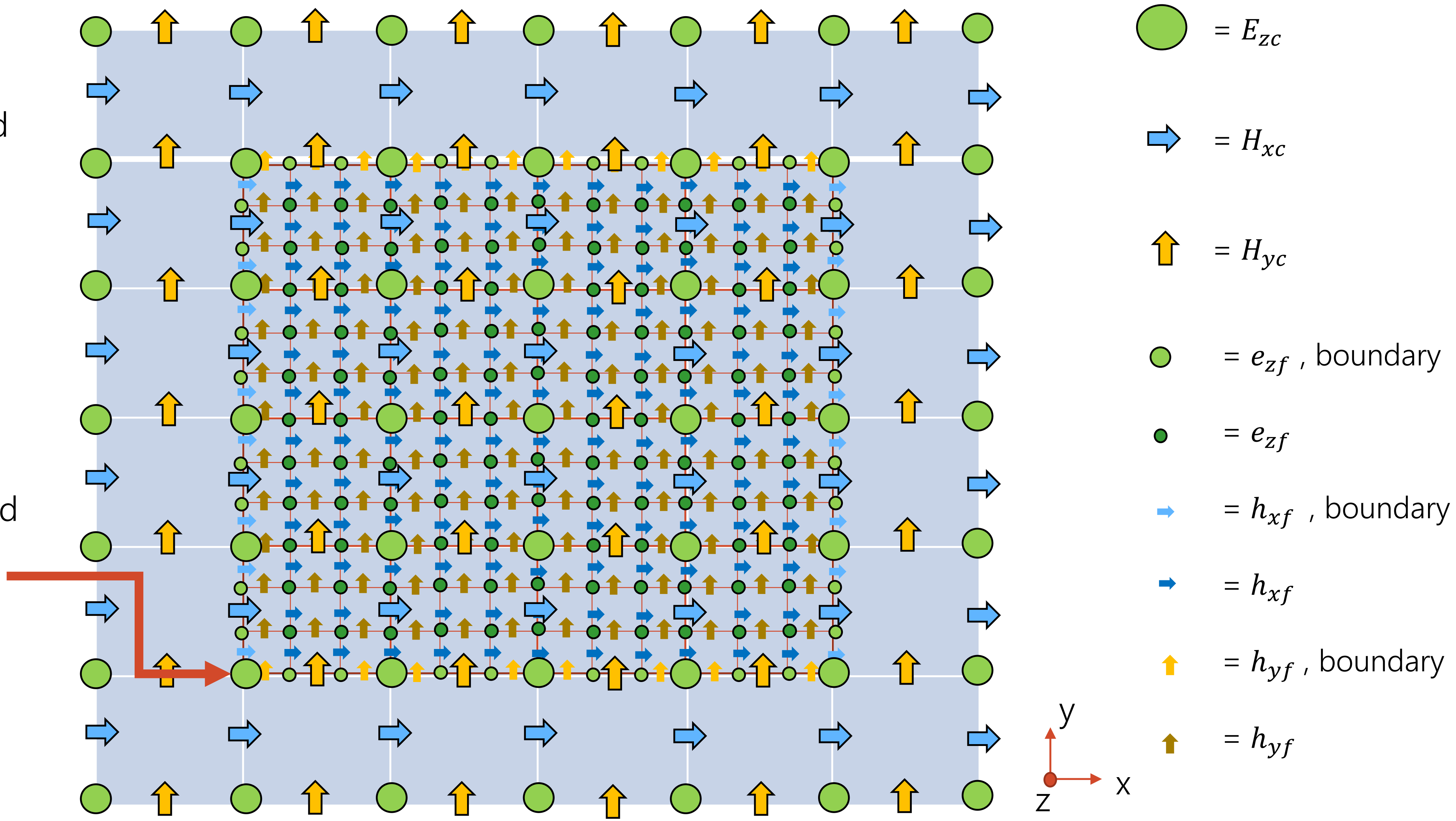
[1] S. Wang, "Numerical examinations of the stability of FDTD subgridding schemes," ACES Journal, vol. 22, no. 2, Jul. 2007.
[2] F. L. Teixeira, "A summary review on 25 years of progress and future challenges in FDTD and FETD techniques," ACES Journal, vol. 25, no. 1, Jan. 2010.
[3] M. F. Hadi and R. K. Dib, "Eliminating interface reflections in hybrid low-dispersion FDTD algorithms," ACES Journal, vol. 22, no. 3, Nov. 2007.
[4] J. Nehrbass and R. Lee, "Optimal finite-difference sub-gridding techniques applied to the Helmholtz equation," IEEE Transactions on Microwave Theory and Techniques, vol. 48, no. 6, pp. 976–984, Jun. 2000.

Superimposed Coarse and Fine Grid – TMz Case

(i_c, j_c) = coarse grid indices

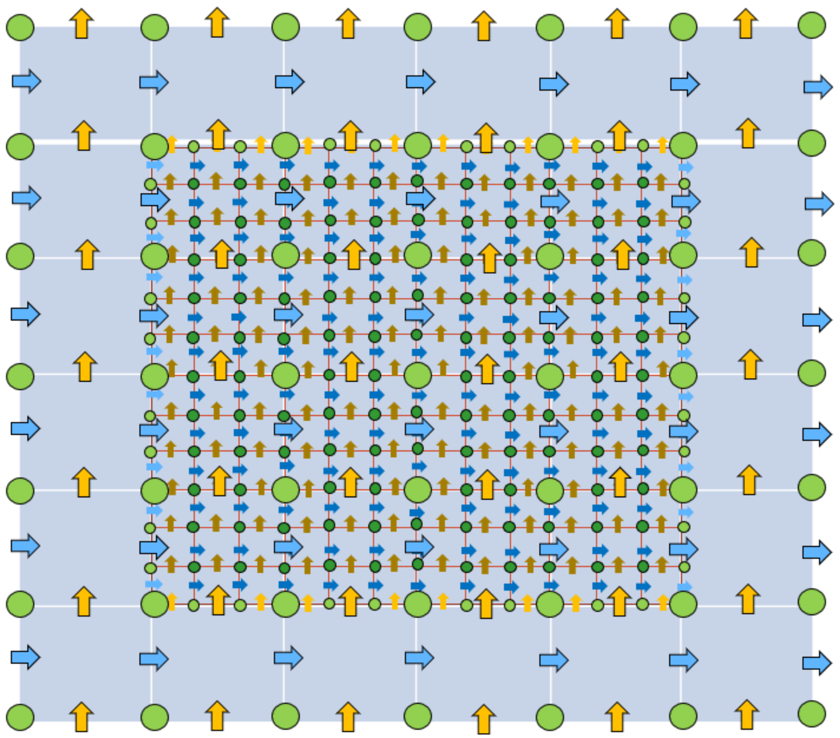
(i_f, j_f) = fine grid indices

(I_f, J_f) = coarse grid relationship to beginning of fine grid



FDTD Subgridding Process

1. Update H_{xc} everywhere in the coarse grid.
2. Update h_{xf} everywhere in the fine grid using updating equation.
3. Update only boundary H_{xc} with the new value for h_{xf} at specific collocated locations.
4. Update H_{yc} everywhere in the coarse grid.
5. Update h_{yf} everywhere in the fine grid using updating equation.
6. Update only boundary H_{yc} with the new value for h_{yf} at specific collocated locations.
7. Update E_{zc} everywhere in the coarse grid.
8. Update the collocated e_{zf} with the value of E_{zc}.
9. Interpolation of non-collocated e_{zf}
10. Update non-boundary e_{zf} using fine grid magnetic fields.
11. Repeat steps 1-10 for all following time steps.



Step 9 - e_{zf} , boundary interpolation method

$\boldsymbol{N_{xc} = N_{yc} = 6}$ $\boldsymbol{n_{xf} = n_{yf} = 12}$

9. Update e_{zf} only along boundary using interpolation of E_{zc}.
   a) Corner & Edge boundaries
      i. Interpolate between two closest E_{zc} coarse nodes.
      ii. Fine grid nodes, e_{zf}, will receive 2/3 the value of the node closest (1 fine grid step away) and it will receive 2/3 of the next closest coarse node (2 fine grid steps away). Equation 1.

$$e_{zf}^{n+1}(I_f + i_f, J_f + j_f) = \frac{fine\ grid\ steps\ to\ \mathbf{closest}\ coarse\ node}{3} E_{zc}^{n+1}(I_f + i_f, J_f + j_f)$$

$$+ \frac{fine\ grid\ steps\ to\ \mathbf{next\ closest}\ coarse\ node}{3} E_{zc}^{n+1}(I_f + i_f, J_f + j_f) \quad (1),$$

where,
(I_f, J_f) = index for the beginning of the fine grid in terms of the coarse grid coordinates,
(i_f, j_f) = indices of fine grid component locations within the fine grid

Chevalier, Luebbers, and Cable, "FDTD Local Grid with Material Traverse," IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATION, VOL. 45, NO. 3, MARCH 1997

2D Subgridding – Single Subgrid Region

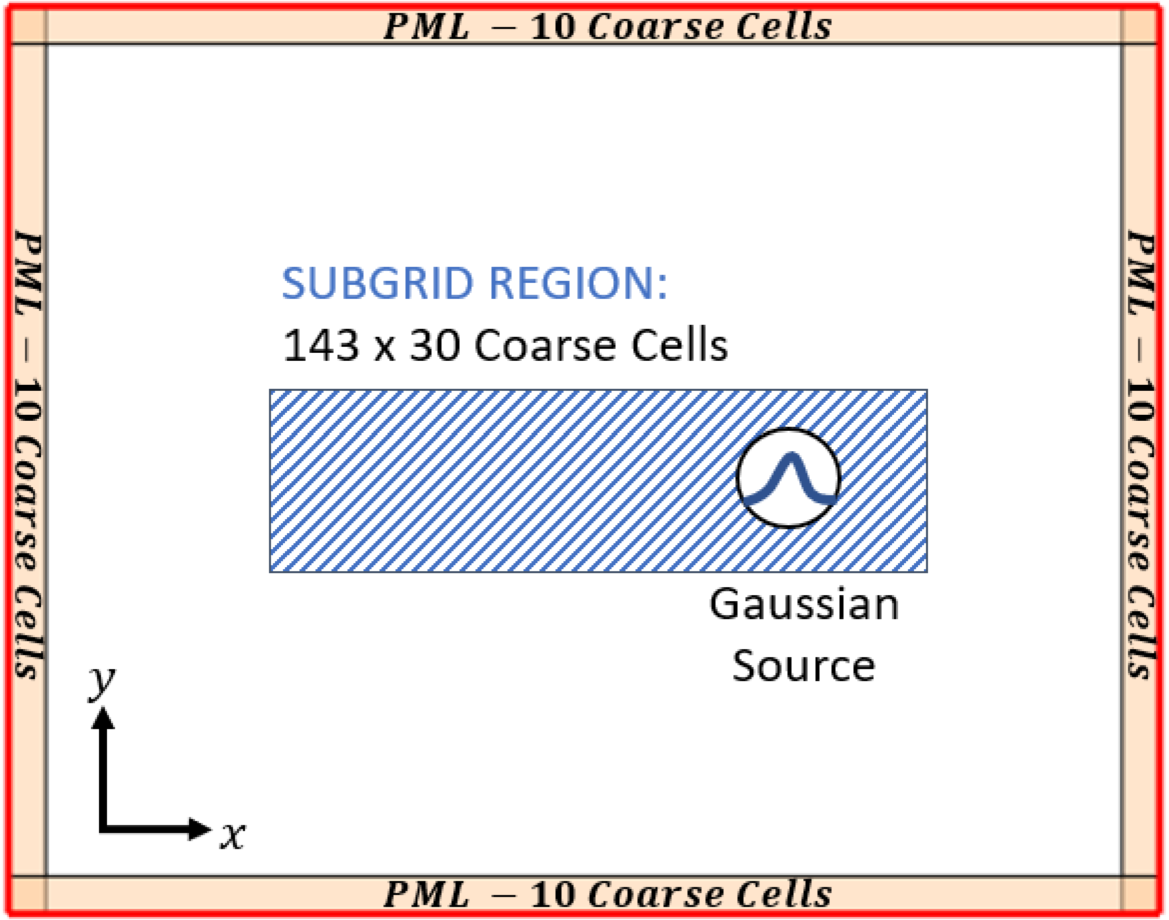
Normalized Difference 2D Calculation Equation

$$Max\ Normalized\ Difference(i,j,t) = \frac{\left|E_{zSubgrid}(i,j,t)\ - E_{zReference}(i,j,t)\right|_{Max}}{\left|E_{zReference}(i,j,t)\right|_{Max}} x100$$

$$for$$
$$t = all\ time$$
$$i = [1{:}\, nx + 1]$$
$$j = [1{:}\, ny + 1]$$

Problem Space

2D Domain: 308 x 243 Coarse Cells



Contrast Ratio	Coarse Cell Size ($dx = dy$)	Fine Cell Size ($dx_{fine} = dy_{fine}$)	Time Step Size (dt)	Number of Time Steps
1:3	$3\ mm$	$1\ mm$	$2.1\ ps$	3,000
1:9	$3\ mm$	$0.33\ mm$	$0.7\ ps$	9,000
1:15	$3\ mm$	$0.2\ mm$	$0.42\ ps$	15,000
1:27	$3\ mm$	$0.11\ mm$	$0.24\ ps$	27,000

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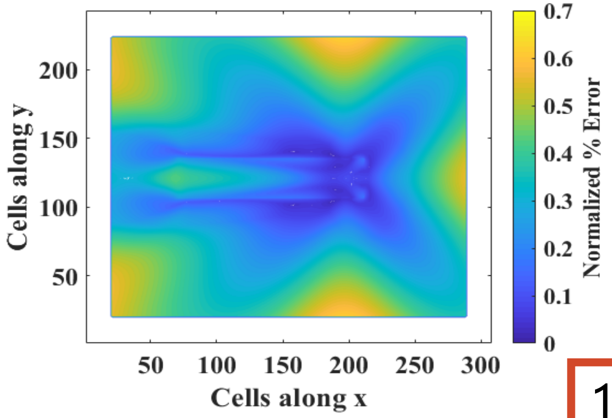
S22 vs. Hybrid Results (Contrast Ratio = 1:3, 1:9, 1:15, 1:27)

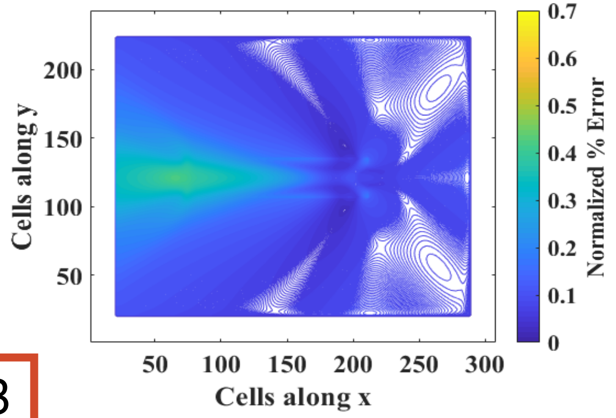
1:3

S22 Error



Maximum S22 Error: 0.62%

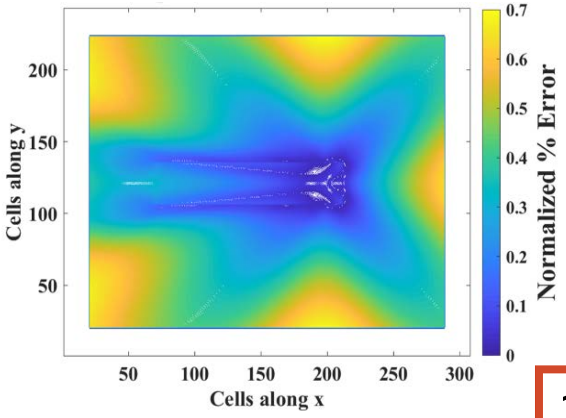
Hybrid Error



Maximum Hybrid Error: 0.42%

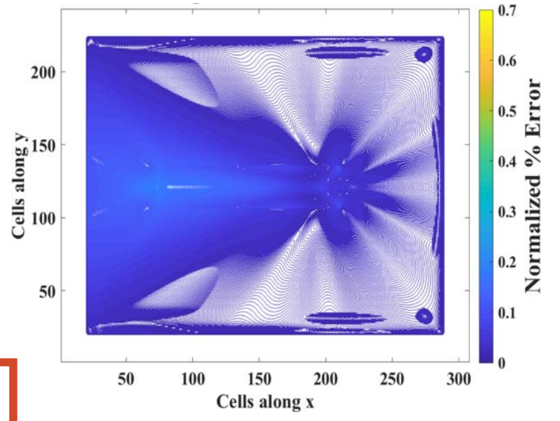
1:9

S22



Maximum S22 Error: 0.69%

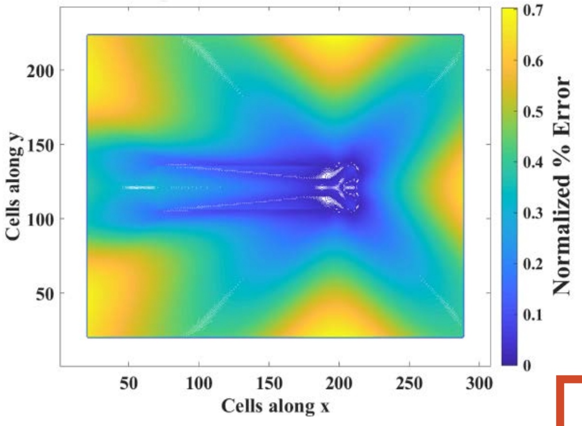
Hybrid Error



Maximum Hybrid Error: 0.18%

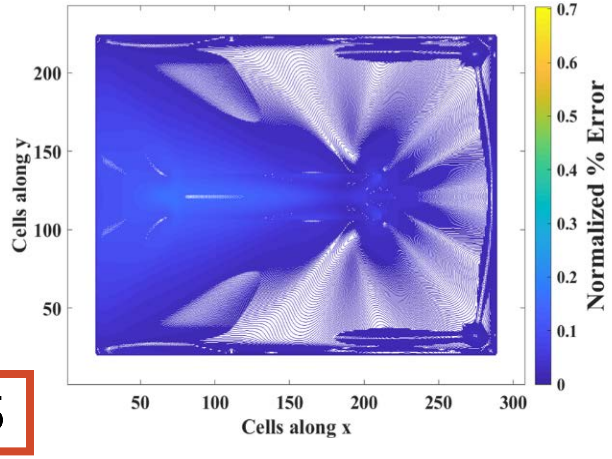
1:15

S22 Error



Maximum S22 Error: 0.70%

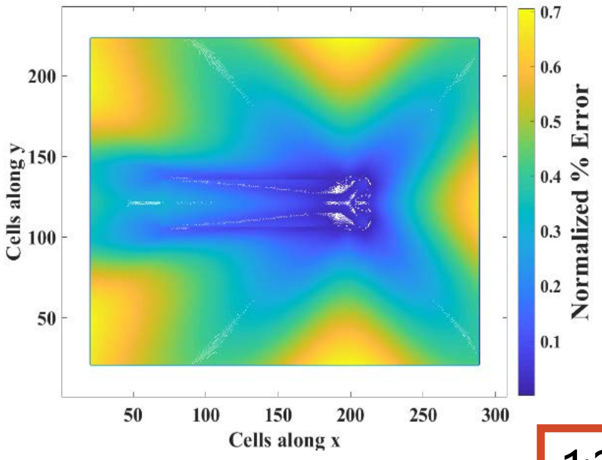
Hybrid Error



Maximum Hybrid Error: 0.16%

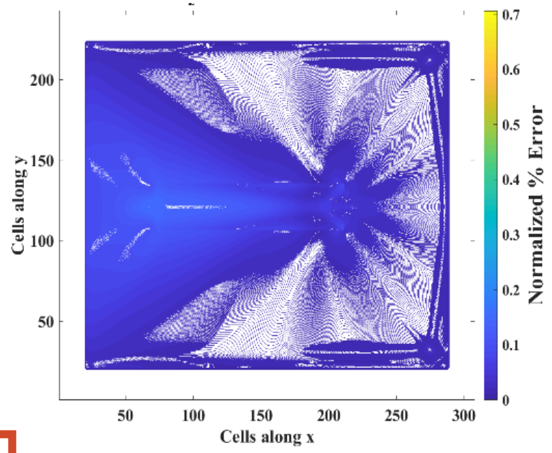
1:27

S22



Maximum S22 Error: 0.71%

Hybrid Error



Maximum Hybrid Error: 0.15%

S22 vs. Hybrid Error Comparison

Contrast Ratio	$S22$ Maximum % Error $\frac{max\lvert E_z(i,j,t) - E_{z,ref}(i,j,t)\rvert}{\max\lvert E_{z,ref}(i_{source} - i_{offset}, j_{source}, t)\rvert}$	$Hybrid$ $(S24)$ Maximum % Error $\frac{max\lvert E_z(i,j,t) - E_{z,ref}(i,j,t)\rvert}{\max\lvert E_{z,ref}(i_{source} - i_{offset}, j_{source}, t)\rvert}$	Hybrid Improvement $\frac{\lvert Error_{S22} - Error_{S24}\rvert}{\lvert Error_{S22}\rvert}$
1:3	0.6168 %	0.4202 %	*32 %*
1:9	0.6971 %	0.1803 %	*74 %*
1:15	0.7036 %	0.1614 %	*77 %*
1:27	0.7061 %	0.1550 %	*78 %*

Total CPU Times

Contrast Ratio	Reference	S22	Hybrid	Hybrid Improvement
	Total Time	Total Time	Total Time	Saved Time
1:3	0.331	0.050	0.054	***83.69%***
1:9	7.780	0.326	0.316	***95.94%***
1:15	34.658	1.371	1.373	***96.04%***
1:27	3052.957	491.586	495.241	***83.78%***

**All simulations were run using MATLAB R2018a software on a 64-bit Intel® Xeon® CPU E5-2680 0 at 2.70 GHz, 2.70 GHz (2 processors) with 256 GB of RAM.*

Memory Usage Breakdown

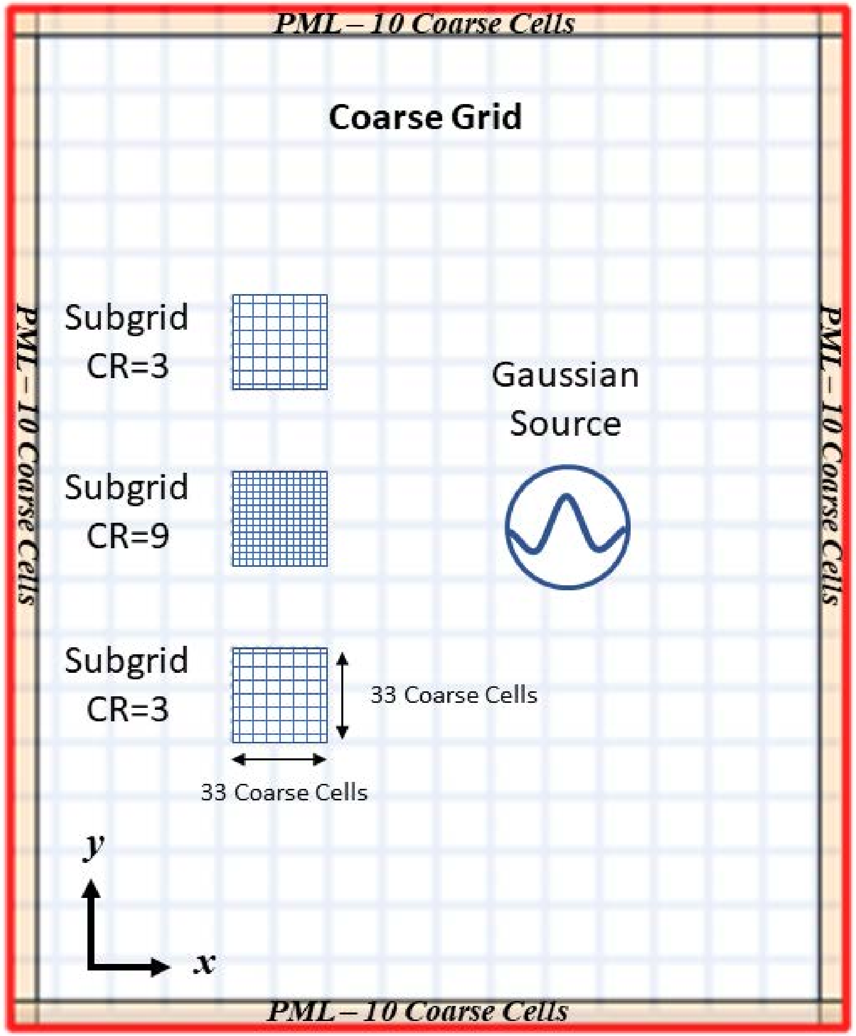
Contrast Ratio	Memory (GB)			Hybrid Improvement
	Reference	*S22*	*Hybrid*	*Memory Saved*
1:3	2.371	2.154	2.156	**9.07%**
1:9	4.294	2.230	2.232	**48.02%**
1:15	17.055	2.327	2.327	**86.36%**
1:27	21.192	2.581	2.543	**88.47%**

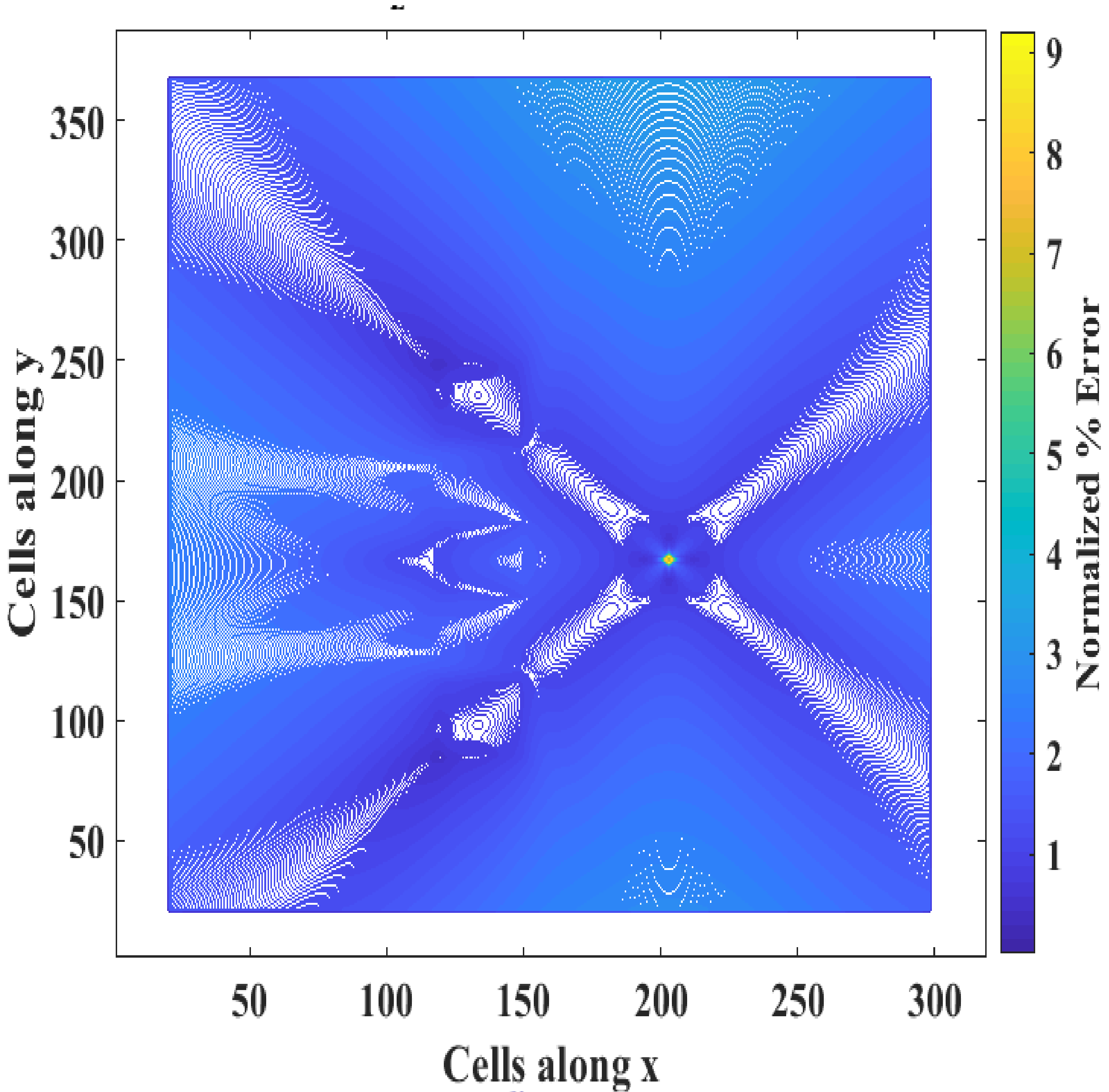
**All simulations were run using MATLAB R2018a software on a 64-bit Intel® Xeon® CPU E5-2680 0 at 2.70 GHz, 2.70 GHz (2 processors) with 256 GB of RAM.*

2D Subgridding – Multiple Subgrid Regions

Hybrid FDTD Problem Space & Results

2D Domain: 319 x 388 Coarse Cells





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Research Achievements & Future Work

Achievements:

- Acceptable levels of error in the S22 and Hybrid domains.
- Acceptable levels of error with higher contrast ratios, up to 27.
- Significant speedup in CPU time utilizing subgridding methods.
- Significant reduction in memory usage.
- Successful implementation of multiple subgrid regions in a 2D domain.

Future Work:

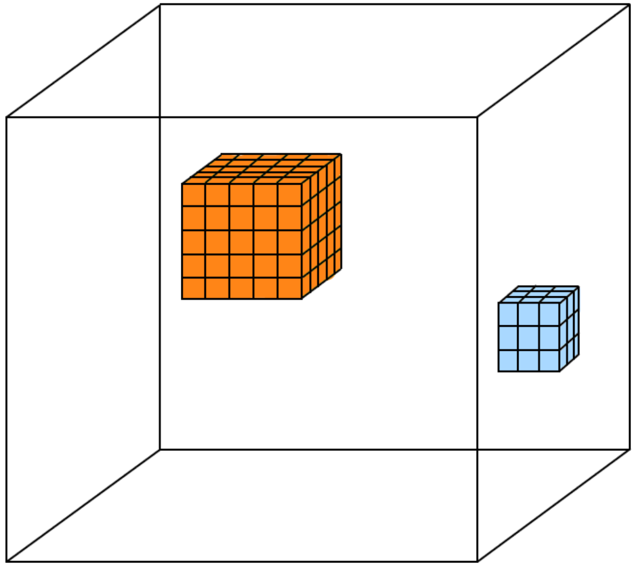
- Integrating subgridding in a 3D computational domain to begin testing on realistic scenarios such as filter and antenna array problems.
- Implementing a hybrid 4th and 2nd order FDTD calculation of the electric and magnetic fields to increase accuracy of the simulations in the coarse domain.
- Subgrid Regions with higher contrast ratios, 30, 90, etc.

Publications

- M. Le, M. Hadi, and A. Elsherbeni, "Quantifying sub-gridding errors when modeling multiscale structures with FDTD," 2019 International Applied Computational Electromagnetics Society (ACES), Miami, FL, USA, pp. 1-2, 2019.

- M. Le, M. Hadi, and A. Elsherbeni, "Quantifying Sub-gridding Errors in Standard and Hybrid Higher Order 2D FDTD Simulations," 2020 International Applied Computational Electromagnetics Society (ACES), Monterey, CA, USA, pp. 1-2, 2020.

- Submitted: M. Le, M. Hadi, and A. Elsherbeni, "Quantifying Subgridding Errors in FDTD Method with Second and Fourth Order Derivative Approximations", ACES Journal Papers, April 2020.



Achievements: ACES 2019 Student Paper Competition 3rd Place Winner, 2019 ACES Conference, Miami, FL.

Questions?

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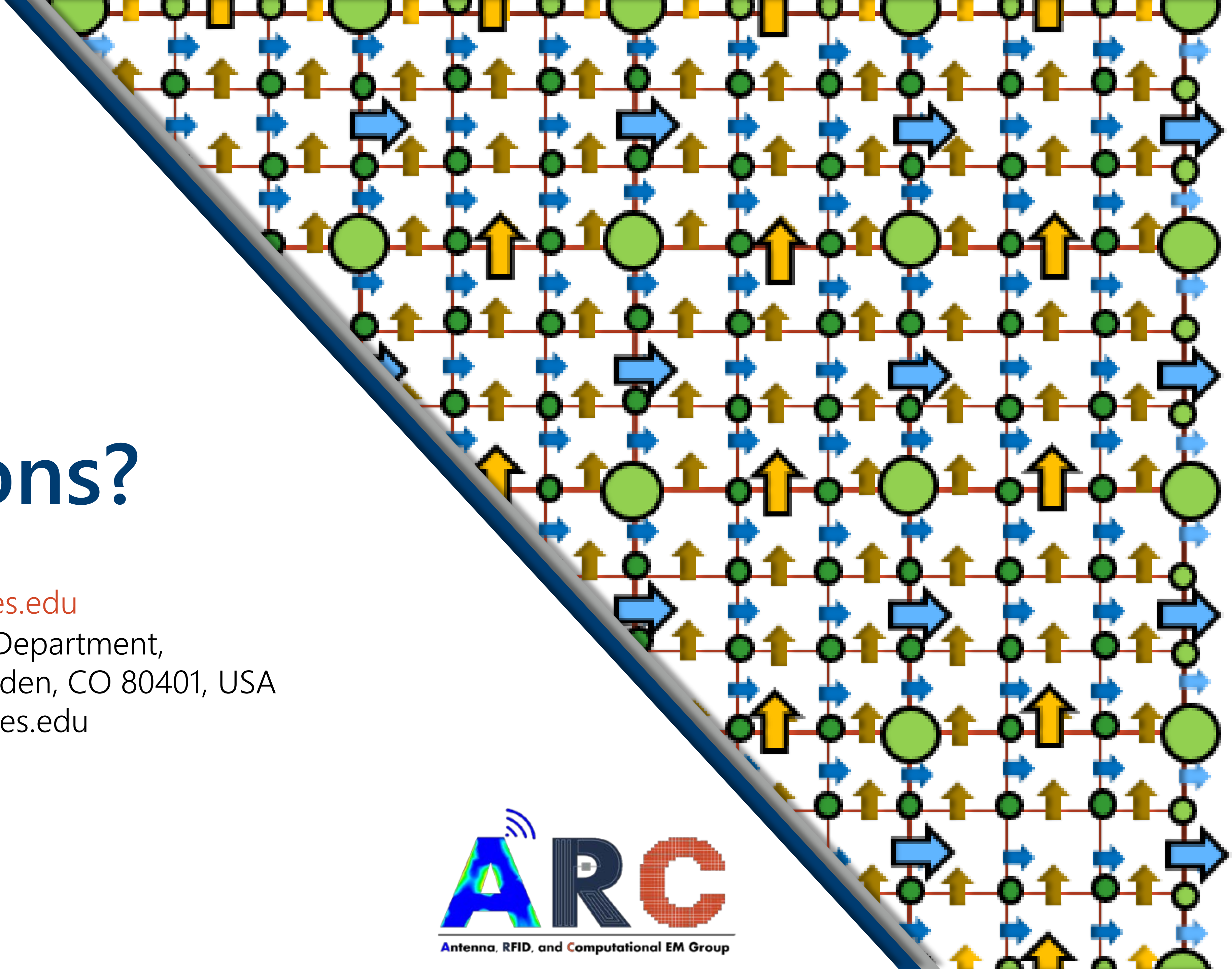

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FDTD Subgridding References

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3. *S. V. Georgakopoulos, C. A. Balanis, C. R. Birtcher, and R. A. Renaut, "HIRF Penetration and PED Coupling Analysis for Scaled Fuselage Models Using a Hybrid Subgrid FDTD(2,2)/FDTD(2,4) Method", IEEE Transactions on Electromagnetic Compatibility, vol. 45, pp. 293-305, May. 2003*
4. *S. V. Georgakopoulos, "Higher-Order Finite Difference Methods for Electromagnetic Radiation and Penetration." Order No. 3031456, Arizona State University, Ann Arbor, 2001.*
5. *S. V. Georgakopoulos, C. A. Balanis, C. R. Birtcher and R. A. Renaut, "Higher-Order Finite Difference Schemes for Electromagnetic Radiation, Scattering, and Penetration, Part II: Applications", IEEE Antennas and Propagation Magazine, vol. 44, pp. 92-101, April 2002*
6. *S. V. Georgakopoulos, C. A. Balanis, C. R. Birtcher and R. A. Renaut, "Higher-Order Finite Difference Schemes for Electromagnetic Radiation, Scattering, and Penetration, Part I: Theory", IEEE Antennas and Propagation Magazine, vol. 44, pp. 134-142, Feb. 2002*
7. *S. V. Georgakopoulos, R. A. Renaut, C. A. Balanis and C. R. Birtcher, "A Hybrid Fourth-Order FDTD Utilizing a Second-Order FDTD Subgrid", IEEE Microwave and Wireless Components Letters, vol. 11, pp. 462-464, Nov. 2001*